# A Biased-Randomised Large Neighbourhood Search for the Two-Dimensional Vehicle Routing Problem with Backhauls

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**Abstract**

The two-dimensional loading vehicle routing problem with clustered backhauls (2L-VRPB) is a realistic extension of the classical vehicle routing problem where both delivery and pickup demands are composed of non-stackable items. Despite the fact that the 2L-VRPB can be frequently found in real-life transportation activities, it has not been analysed so far in the literature. This paper presents a hybrid algorithm that integrates biased-randomised versions of vehicle routing and packing heuristics within a Large Neighbourhood Search metaheuristic framework. The use of biased randomisation techniques allows to better guide the local search process. The proposed approach for solving the 2L-VRPB is tested on an extensive set of instances, which have been adapted from existing benchmarks for the two-dimensional loading vehicle routing problem (2L-VRP). Additionally, when no backhauls are considered our algorithm is able to find new best solutions for several 2L-VRP benchmark instances with sequential oriented loading, both with and without items rotation.

*Keywords:* Metaheuristics, Packing, Routing, Transportation, Vehicle routing problem

## 1. Introduction

The Vehicle Routing Problem (VRP) is a well-known combinatorial optimisation problem in which a fleet of vehicles has to service a set of customers at

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the lowest possible cost [1, 2]. The most basic variant of the VRP is the Capacitated Vehicle Routing Problem (CVRP), and more complex versions are built upon it. In the CVRP, it is assumed that there is an homogeneous fleet of vehicles with restricted capacity, based at a central depot, which is used to satisfy customers' demands by visiting them only once. Additional restrictions, such as distance or time-based constraints, are often considered in richer variants of the problem. The CVRP and richer versions have been extensively studied due to their potential applicability on real-life transportation activities [3, 4]. Moreover, the CVRP is regarded as a NP-Hard problem, and so are its extensions. Hence, it constitutes a challenging problem and a rich environment to develop new methods, either exact or approximate [5, 6].

In this paper, we consider a realistic variant of the CVRP that combines vehicle routing and loading (packing) aspects as well as backhauls. This variant is an extension of the Two-dimensional Capacitated Vehicle Routing Problem (2L-VRP) [7], where customers' demands consist of a set of rectangular items that cannot be stacked due to their weight, dimensions, or fragility. Our work was originally motivated by real-life transportation activities at Opein (www.opein.com), a medium-sized company which provides industrial equipment to its customers, mostly in the building-construction field. Opein has to periodically deliver and pickup a large variety of industrial machinery, namely aerial-work platforms, energy-generation sets, compressors, dumpers, forklifts and professional cleaning equipment. Similar issues arise in other industries where large-sized items pickup and delivery is also required, e.g. furniture or appliances. These items must be efficiently packed on the truck surface to attain a high vehicle's utilisation. Thus, one needs to consider not only the items weight, but also their length and width. For the purposes of this paper, we consider these items to be of rectangular shape, and we assume they cannot be piled up or overlap.

Several variants of the 2L-VRP have been defined, depending on constraints on the loading configuration: *(i)* oriented loading (OL), where items cannot be rotated; *(ii)* non-oriented or rotated loading (RL), allowing items to be rotated 90° when loaded on the vehicle; *(iii)* sequential loading (SL), where items should be loaded in reverse order to the customers' visits, as they cannot be rearranged inside the vehicle once the route has started; and *(iv)* unrestricted loading (UL), allowing items rearrangement during the distribution process. The different combinations of the aforementioned constraints yield a classification into four 2L-VRP variants: *(i)* two-dimensional sequential oriented loading ($2|SO|L$); *(ii)* two-dimensional sequential non-oriented (rotated) loading ($2|SR|L$); *(iii)* two-dimensional unrestricted oriented loading ($2|UO|L$); and *(iv)* two-dimensional unrestricted non-oriented (rotated) loading ($2|UR|L$). So far, only Fuellerer et al. [8] have solved all four problem variants. In fact, only Fuellerer et al. [8] and Dominguez et al. [9] have addressed the non-oriented loading configurations (see Section 3). In this work, we consider both oriented and non-oriented cases combined with sequential loading ($2|SO|L$ and $2|SR|L$, respectively). Sequential loading might be a frequent requirement in real-life distribution practices, since unloading and re-loading heavy machinery might represent a significant cost in terms of both time and resources. In our opinion, allowing items rotation is also a realistic assumption that has shown to attain significant savings when compared to oriented configurations [9].

As an industrial machinery hire company, Opein also needs to fetch their equipment at the end of the hiring lease. These adds a considerable burden to their logistics and yields separate problems for delivery and pickups, dealing with spatial and capacity constraints in both cases. Backhauling has been proven to be an efficient way to achieving significant savings [10]. In the Vehicle Routing Problem with Backhauls (VRPB), the set of customers is divided into delivery locations (*linehaul*) or pickup points (*backhaul*). The critical assumption is that all deliveries must be made on each route before any pickups can be made (i.e. clustered backhauls). This arises from the fact that the vehicles are rear-loaded, and rearrangement of the loads on the trucks at the delivery points is not deemed economical or feasible [11]. This is also coherent with our selection of sequential loading variants of the 2L-VRP. In addition, no route can contain only backhaul customers, although linehaul-only routes are allowed. Figure 1 shows an example of a route containing linehaul and backhaul customers and its corresponding packing plans. Notice that items have been loaded on the vehicle in inverse order to the visits for linehaul customers, while items picked up at backhaul locations are loaded in sequential order.

This article proposes a hybrid algorithm for solving the Two-dimensional Capacitated Vehicle Routing Problem with clustered Backhauls (2L-VRPB), considering sequential loading and items rotation. To the best of our knowledge, this is the first time this problem is tackled in the literature. Only Malapert et al. [12] have previously studied the combination of both problems, but without considering items rotation. Our proposed method combines a Large Neighbourhood Search (LNS) metaheuristic framework [13] with biased-randomised versions of classical routing and packing heuristics. Biased randomisation of heuristics refers to the use of skewed probability distributions to induce an oriented (i.e. biased) random behaviour of the heuristic, transforming a deterministic method into a probabilistic algorithm often suitable for multi-start and parallel approaches [14]. In our case, we use a biased-randomised version of the well-known Clarke and Wright savings heuristic for the CVRP [15], enhanced with memory-based techniques [16]. In order to deal with backhauls, we omit all edges connecting backhaul locations to linehaul customers, so a vehicle is not allowed to perform any delivery once it visits the first backhaul customer in its route. We integrate the packing process within our randomised savings-based heuristic. To compute packing plans, we use biased-randomised versions of the Best-Fit heuristic [17] and the Touching Perimeter algorithm [18]. This process is used to check feasibility before merging any two routes in our routing heuristic. Our results show that the proposed approach is an efficient way of solving the 2L-VRPB with oriented and non-oriented loading, as we show in Section 5. In addition, our method has been able to generate new best-known solutions for benchmark instances of the 2L-VRP with sequential loading, both with and without rotation ($2|SR|L$ and $2|SO|L$, respectively).

The remaining of this paper is structured as follows. We first formally intro-

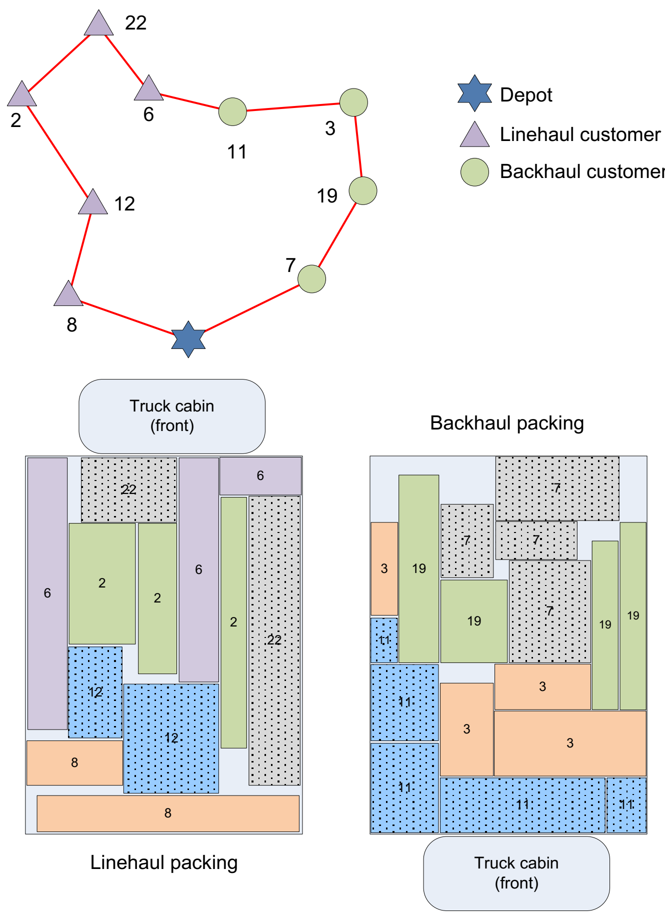


Figure 1: Example of a combined linehaul-backhaul route with two packing plans.

duce the 2L-VRPB problem in detail in Section 2 and provide an overview on related work in Section 3. A detailed description of our Large Neighbourhood Search with biased randomisation approach is given in Section 4. Section 5 describes some numerical experiments that contribute to illustrate and validate our approach. Finally, Section 6 summarises the main contributions and results of this work.

## 2. Problem Definition

In the 2L-VRPB a complete undirected graph $G = (V, E)$ is given, where $V = \{0\} \cup L \cup B$ is a set of $1 + n + m$ vertices: the vertex 0 corresponds to the depot, while disjoint subsets $L = \{1, \dots, n\}$ and $B = \{n+1, \dots, n+m\}$ represent linehaul and backhaul customers, respectively. For simplicity of notation, we design the set of customers vertices $V \setminus \{0\}$ as $V_0$. The edge set $E = \{(i,j) \mid i,j \in V;\ i \neq j\}$ connects customers with each other and with the depot. Each of these edges has an associated distance-based cost corresponding to the travelling distance $c_{ij} > 0$ between nodes $i$ and $j$. We assume costs are symmetric, i.e. $c_{ij} = c_{ji}$.

Transportation of goods is performed by a fleet of $K$ identical vehicles, initially located at the depot, each with maximum weight-loading capacity $Q$ and a loading area $A = W \times H$. For each customer $i$ ($i \in V_0$), there are $m_i > 0$ items with a total weight $q_i$ to be delivered (linehaul) or collected (backhaul). It is assumed that the depot has no demand, i.e. $m_0 = 0$. For each customer's item, its length and width dimensions are denoted by $h_{il}$ and $w_{il}$ ($1 \leq l \leq m_i$), respectively. Thus, the total area of the items of customer $i$ can be denoted $a_i = \sum_{l=1}^{m_i} w_{il} h_{il}$.

The resolution of the 2L-VRPB consists in finding a set of routes that minimise the total cost (i.e. travelled distance), fulfilling the following constraints:

(i) Every route starts and finishes at the depot.
(ii) All routes include at least one linehaul customer, i.e. routes containing only backhaul customers are not allowed.
(iii) Customers requiring a delivery (linehaul) must be serviced before customers requiring a pickup (backhaul).
(iv) Each customer, linehaul or backhaul, is visited exactly once. For this reason, all items demanded (linehaul) or supplied (backhaul) by each customer should fit into a single vehicle (item clustering).
(v) In each route, neither items delivered to linehaul customers nor items picked up from backhaul customers surpass vehicle's capacity and loading surface area.

The routing aspects of the problem can be formulated as follows:

$$\min \sum_{k \in K} \sum_{\substack{i,j \in V \\ i \neq j}} c_{ij} z_{ijk} \tag{1}$$

subject to:

$$\sum_{j \in L} z_{0jk} = \sum_{i \in V_0} z_{i0k} \quad \forall k \in K \tag{2}$$

$$\sum_{k \in K} \sum_{i \in V} z_{ijk} = 1 \quad \forall j \in V_0 \tag{3}$$

$$\sum_{i \in V} z_{iuk} = \sum_{j \in V} z_{ujk} \quad \forall u \in V_0, \forall k \in K \tag{4}$$

$$\sum_{\substack{i \in B \\ j \in L}} z_{ijk} = 0 \quad \forall k \in K \tag{5}$$

$$\sum_{\substack{i \in L \\ j \in B}} z_{ijk} \leq 1 \quad \forall k \in K \tag{6}$$

$$\sum_{k \in K} \sum_{j \in V_0} z_{0jk} \leq K \tag{7}$$

$$\sum_{\substack{i \in L \\ j \in V}} q_i z_{ijk} \leq Q \quad \forall k \in K \tag{8}$$

$$\sum_{\substack{i \in L \\ j \in V}} a_i z_{ijk} \leq A \quad \forall k \in K \tag{9}$$

$$\sum_{\substack{i \in B \\ j \in V}} q_i z_{ijk} \leq Q \quad \forall k \in K \tag{10}$$

$$\sum_{\substack{i \in B \\ j \in V}} a_i z_{ijk} \leq A \quad \forall k \in K \tag{11}$$

$$z_{ijk} \in \{0, 1\} \quad \forall i, j \in V, i \neq j, \forall k \in K \tag{12}$$

The objective function (1) minimises the total distance cost required to service all customers. Constraint (2) ensures that the number of vehicles departing the depot is the same as the number of vehicles returning to it. Notice that routes are only allowed to start with a linehaul customer. On the other hand, vehicles can return to the depot either from a linehaul or backhaul customer, allowing for linehaul-only routes. Equations (3) and (4) ensure that each customer is visited exactly once and that if a vehicle visits a customer, it must also depart from it. Constraints (5) and (6) enforce that no linehaul customers are visited after servicing any backhaul customer and that visits are clustered, allowing at most one edge per route connecting linehaul to backhaul locations. Equation (7) imposes that the maximum number of available vehicles is never exceeded. Finally, constraints (8) and (9) ensure that the maximum capacity and loading area of the vehicle are never exceeded at linehaul customers. Equations (10) and (11) are the equivalent restrictions for backhaul locations. Notice that, because we assume vehicles depart the depot loaded and deliver all the carried goods before visiting any backhaul customer –due to the clustered backhauls assumption–, we can enforce capacity and loading area constraints separately on each part of the route.

In addition, we need to take into account several considerations to regard a packing as feasible:

(i) Every item must be loaded without overlapping and with its edges parallel to the edges of the vehicle (orthogonal loading).

(ii) All items associated to any given customer must be loaded and unloaded from the rear of the vehicle, employing only straight movements (one per item). Items are not allowed to be rearranged at customer sites (sequential loading constraint).

(iii) Depending on the loading configuration, items cannot be rotated (oriented loading $2|SO|L$) or, on the contrary, a 90° rotation is allowed (non-oriented loading $2|SR|L$).

We start by defining the loading surface of a vehicle as a $W \times H$ matrix with indexes $x \in \{1, \ldots, W\}$ and $y \in \{1, \ldots, H\}$. Thus, the coordinates $(x_{ilk}, y_{ilk})$ represent, for vehicle $k$ delivering (collecting) product $l$ to (from) customer $i$, the position of the bottom-left corner of item $l$ on the vehicle's loading surface.

We define a feasible route $R^k$ as the subset of customers ($R^k \subseteq V_0$) visited by vehicle $k$. Due to the clustered visits assumption, we can separate the route $R^k$ into two separate subsets for linehaul ($R^k_L$) and backhaul ($R^k_B$) customers, i.e. $R^k = R^k_L \cup R^k_B$ and $R^k_L \cap R^k_B = \emptyset$. We denote the order of the visits in a particular route $R^k$ by a bijection $\sigma^k : R^k \to \{1, \ldots, |R^k|\}$. Since visits are clustered, we can define a separate bijection for each part of the route:

$$\sigma^k_L : R^k_L \to \{1, \ldots, |R^k_L|\}$$
$$\sigma^k_B : R^k_B \to \{|R^k_L| + 1, \ldots, |R^k|\}$$

Depending on the characteristics of the problem, items may be loaded in the order visits are performed (pick-up) or in inverse order (delivery). However, whenever $(R^k, \sigma^k)$ is a feasible route, then also its inverse $(R^k, \sigma^{kR})$ is a feasible route with an equivalent loading pattern that can be obtained by applying a symmetry operation [7]. This result, combined with the separability of the route's linehaul-backhaul subsets, allows us to generalise the packing constraints to each subset, and simplify the notation. Hence, we will denote by $R$ a subset of customers forming a route or partial route –i.e. linehaul or backhaul subsets–, with a sorting denoted by a bijection $\sigma : R \to \{1, \ldots, |R|\}$.

Additionally, we define the variable $\Omega_{il}$ to indicate if item $l$ of customer $i$ has been rotated ($\Omega_{il} = 1$) or not ($\Omega_{il} = 0$). Using the introduced variables and notation, we can define the packing constraints, adapted from Iori et al. [7], as follows:

$$\begin{aligned} &0 \leq x_{il} \leq (W - w_{il})(1 - \Omega_{il}) + (W - h_{il})\Omega_{il} \quad \wedge \\ &0 \leq y_{il} \leq (H - h_{il})(1 - \Omega_{il}) + (H - w_{il})\Omega_{il} \qquad \forall i \in R, \forall l \in \{1, \ldots, m_i\} \end{aligned} \tag{13}$$

$$\begin{aligned}
&x_{il} + w_{il}(1-\Omega_{il}) + h_{il}\Omega_{il} \le x_{jl'} && \vee\\
&x_{jl'} + w_{jl'}(1-\Omega_{jl'}) + h_{jl'}\Omega_{jl'} \le x_{il} && \vee\\
&y_{il} + h_{il}(1-\Omega_{il}) + w_{il}\Omega_{il} \le y_{jl'} && \vee\\
&y_{jl'} + h_{jl'}(1-\Omega_{jl'}) + w_{jl'}\Omega_{jl'} \le y_{il}\\
&\forall i,j \in R, \forall l \in \{1,\ldots,m_i\}, \forall l' \in \{1,\ldots,m_j\}, (i,l) \neq (j,l') && (14)
\end{aligned}$$

$$\begin{aligned}
&y_{il} \ge y_{jl'} + h_{jl'}(1-\Omega_{jl'}) + w_{jl'}\Omega_{jl'} && \vee\\
&x_{il} + w_{il}(1-\Omega_{il}) + h_{il}\Omega_{il} \le x_{jl'} && \vee\\
&x_{jl'} + w_{jl'}(1-\Omega_{jl'}) + h_{jl'}\Omega_{jl'} \le x_{il}\\
&\forall i,j \in R : \sigma(i) < \sigma(j), \forall l \in \{1,\ldots,m_i\}, \forall l' \in \{1,\ldots,m_j\} && (15)
\end{aligned}$$

Constraint (13) ensures that items are loaded within the loading surface. Expression (14) permits avoiding any two items overlapping on the surface of the vehicle. Finally, constraint (15) ensures that items can be loaded/unloaded without any item interfering on their straight movement from their assigned position to the loading/unloading side of the vehicle.

## 3. Literature Review

In recent years, VRP variants combining classical and real-life constraints have attracted an increasing interest from the research community [3, 4], some of them even including environmental issues [19, 20, 21]. The 2L-VRPB integrates two combinatorial optimisation problems: the Two-dimensional Vehicle Routing Problem (2L-VRP) and the Vehicle Routing Problem with clustered Backhauls (VRPB). To the best of our knowledge, only a previous study by Malapert et al. [12] analyses the combination of both problems. They propose a Constraint Programming model that integrates both the routing and packing components based on a scheduling approach. However, this model does not allow for items rotation during the loading stage. Moreover, the authors do not report any computational results.

Both VRPB and 2L-VRP have received far more attention separately. Several exact and heuristic methods have been proposed to solve the VRPB. Parragh et al. [22] provide a comprehensive survey on approaches to tackle Pickup and Delivery Problem (PDP) variants, where they include the VRPB as a one of the four sub-classes. Exact approaches perform a systematic search over the solution space, returning the best feasible solution. However, their execution times often relegate their application to small instances. The first exact approach for the VRPB was introduced by Yano et al. [23]. The authors develop a branch-and-bound algorithm based on a set covering approach for a retail chain. They generate optimal routing plans with up to four linehaul and four backhaul customers per route. Toth and Vigo [24] propose a branch-and-bound approach which uses various relaxations to compute lower bounds. The lower bound is progressively strengthened using cutting plane techniques. The developed algorithm generates optimal solutions to most of the VRPB benchmark instances [11, 25]. Mingozzi et al. [26] propose a linear programming formulation and develope a branch-and-bound algorithm. Variable reduction via pricing allows for solving the reduced problem. This approach was able to solve to optimality benchmark instances with up to 100 customers.

The first heuristic approach for the VRPB was proposed by Deif and Bodin [27]. In this paper, the authors develop an extension of the well-known Clarke and Wright savings heuristic [15], capable of solving instances of up to 300 customers. Goetschalckx and Jacobs-Blecha [11] develop a two-phase heuristic. In their method, both clustering and routing phases are solved by means of a space-filling curves approach. Toth and Vigo [25] propose and later extend [28] a cluster-first route-second algorithm. The authors develop a clustering method that exploits information contained within a solution from a relaxed VRPB, followed by a matching and insertion procedure between clusters and an intra-route improving phase.

There exist several metaheuristic implementations for the VRPB. The first metaheuristic approach was proposed by Potvin et al. [29]. Their approach consists of a genetic algorithm combined with a greedy route construction heuristic. Osman and Wassan [30] introduce a reactive tabu search in which tabu periods are dynamically adjusted to control diversification and intensification phases. Later, Wassan [31] combines a reactive tabu search with adaptive memory programming, ensuring faster convergence. The author reported a good number of new best solutions for several benchmark problems. Brandão [32] introduces a new multi-phase tabu search approach. The author applies different procedures to obtain the initial solution based on a K-tree relaxation of the problem. He uses a static strategy, while Osman and Wassan [30] and Wassan [31] use a strategy that is adapted according to the effectiveness of the search. Røpke and Pisinger [33] introduce a unified model for solving many VRP variants, including the VRPB. They transform these different variants into a rich PDP with time windows, which is then solved by means of a large neighbourhood search method enhanced with learning mechanisms. Gajpal and Abad [34] develop an ant colony system with two types of ants: the first type used to allocate customers to vehicles and the second type used to solve the routing problem. Zachariadis and Kiranoudis [35] propose a local search metaheuristic which explores neighbourhoods composed of exchanges of variable-length customer sequences. The authors introduce the concept of promises to diversify the search and eliminate cycling. At each iteration, sequences of customers that appear in the solution are tagged with a promise value based on the solution cost. These promise values are used to discard low-quality moves that involve the same sequences of customers. Palhazi Cuervo et al. [36] introduce an iterated local search algorithm with an oscillating local search heuristic, which explores a wide neighbourhood structure at each iteration and allows transitions between feasible and infeasible regions of the solution space. Recently, García-

Nájera et al. [37] present an evolutionary approach to deal with the VRPB in a multi-objective fashion. The authors define the number of vehicles, total travel cost, and the number of uncollected backhauls as the objectives to be minimised by their approach. They use a VRPB-specific similarity-based selection process to generate solution sets with better coverage of all trade-off possibilities.

The 2L-VRP is a more recent variant which has received considerable attention in the last few years. A survey on VRPs with loading constraints can be found in [38] and [39]. The problem was originally introduced by Iori et al. [7]. The authors propose an exact branch-and-cut algorithm to solve the routing aspects, while they deal with the packing requirements by means of a branch-and-bound algorithm combined with heuristics and effective lower bounds. From the different 2L-VRP subclasses, the authors only tackle the two-dimensional sequential oriented loading case ($2|SO|L$).

Although Iori et al. [7] exact approach is able to find optimal solutions for instances with up to 35 customers and more than 100 items, there are smaller instances where optimality has not been guaranteed yet. For example, Iori and Martello [39] refer to an instance with just 29 customers and 43 items that has yet to be solved optimally. Since these limits are not reasonable for real-life practice, several heuristics and metaheuristics have been proposed for the 2L-VRP. Gendreau et al. [40] propose a tabu search algorithm designed to solve both the sequential ($2|SO|L$) and the unrestricted oriented ($2|UO|L$) loading configurations. The authors report solutions for instances up to 255 customers and 786 items. Zachariadis et al. [41] use a hybrid algorithm combining tabu search and a guided local search. They employ five packing heuristics with different selection criteria to develop feasible loading configurations. Leung et al. [42] present a simulated annealing approach to tackle the oriented versions of the problem. The same authors also develop an extended guided tabu search to deal with the routing component, whereas they introduce a new heuristic for the load configuration checking [43]. Duhamel et al. [44] propose a hybrid approach combining GRASP (Greedy Randomised Adaptive Search Procedure) with evolutionary local search. In their approach, the loading constraint is handled by a resource constraint project scheduling problem heuristic specially tuned to address the bin packing part of the problem. This approach was later embedded in an optimisation framework whose last step was transforming the obtained relaxed solutions into 2L-VRP solutions by solving a dedicated packing problem [45]. Zachariadis et al. [46] present an effective approach named Promise Routing-Memory Packing (PRMP), which combines local search with an effective diversification based on regional aspiration criteria. The loading feasibility of routes is tackled by a packing heuristic enhanced by an innovative memory mechanism. More recently, Wei et al. [47] introduce a variable neighbourhood search algorithm to tackle both sequential and unrestricted loading variants of the 2L-VRP. The authors combine a skyline heuristic with memory-based mechanisms to examine loading constraints.

The oriented versions of the problem have received far more attention in the 2L-VRP literature. The non-oriented (i.e. allowing item rotations) loading configurations have rarely been addressed. To the best of our knowledge, only Fuellerer et al. [8] have solved all four loading configurations. The authors propose an ant colony optimisation algorithm combined with different heuristics dealing with the loading component. Dominguez et al. [9] propose a multi-start biased-randomised algorithm to tackle both two-dimensional unrestricted oriented and non-oriented loading variants. At each restart, a biased randomisation of a savings-based heuristic is combined with an enhanced version of a classical packing heuristic to produce feasible good solutions. Finally, Dominguez et al. [48] also make use of a biased-randomised algorithm to solve the heterogeneous-fleet version of the 2L-VRP with non-oriented loading.

## 4. Biased-Randomised Large Neighbourhood Search

Our method combines a Large Neighbourhood Search (LNS) metaheuristic framework with biased-randomised versions of classical routing and packing heuristics. LNS is a conceptually simple metaheuristic that has proven to be very competitive for solving combinatorial optimisation problems, and particularly routing problems [13]. In this approach, an initial solution is gradually improved by alternately destroying and repairing the solution. A destroy method destructs part of the current solution while a repair method rebuilds the destroyed solution. The destroy method typically contains an element of stochasticity such that different parts of the solution are destroyed in every invocation of the method. The neighbourhood of a solution is then defined as the set of solutions that can be reached by first applying the destroy method and then the repair process. Since the destroy method can destruct a large part of the solution, the neighbourhood contains a large amount of solutions, which explains the name of this metaheuristic framework. Exploring large neighbourhoods facilitates the algorithm to escape from local minima. Typically, LNS requires a reduced number of parameters, thus reducing the need for complex and time-costly tuning processes.

Algorithm 1 presents the pseudo-code of our LNS approach. Biased randomisation (BR) is included at different stages in our method. Biased randomisation of heuristics refers to the use of skewed probability distributions to transform otherwise deterministic methods into probabilistic algorithms [14]. In our BR-LNS approach, we apply biased-randomised techniques on routing and packing heuristics (lines 2 and 15), as well as in the destruction phase (line 13). For the routing component, we used a biased-randomised version of the well-known Clarke and Wright heuristic [15]. As for solving the packing, we use randomised versions of two effective packing heuristics [17, 18]. Our destruction process makes use of splitting techniques similar to those proposed by Juan et al. [16] with additional probabilistic behaviour.

We start our algorithm by generating an initial solution by means of the `Pack-and-Route` procedure (line 2), which is explained in detail later on. This method provides a complete routing solution with packing plans for both linehaul and backhaul customers. We then initialise the best solution so far with our initial solution (line 3) and assign values to variables used to control the destruction operation (lines 4 and 5). The algorithm then starts an iterative

**Algorithm 1** Pseudo-code of our BR-LNS main procedure.

```
1: procedure BR-LNS(inputs, α, β, maxPackIter, t_max)
2:     baseSol ← PACKANDROUTE(inputs, α, β, maxPackIter)
3:     bestSol ← baseSol
4:     improvement ← false
5:     typeSplit ← 0
6:     while time < t_max do
7:         if improvement = false && typeSplit < 3 then
8:             typeSplit ← typeSplit + 1
9:         else if improvement = false && typeSplit = 3 then
10:            typeSplit ← 1
11:        end if
12:        improvement ← false
13:        subSol ← EXTRACTROUTESATRANDOM(baseSol, typeSplit)
14:        subInputs ← EXTRACTNODES(subSol)
15:        newSubSol ← PACKANDROUTE(subInputs, α, β, maxPackIter))
16:        newSol ← UPDATESOL(newSubSol, baseSol)
17:        if cost(newSol) < cost(baseSol) then
18:            improvement ← true
19:            baseSol ← newSol
20:        end if
21:    end while
22:    bestSol ← baseSol
23:    return bestSol
24: end procedure
```

process aiming at improving the best solution by combining partial destruction and reconstruction methods.

The destruction phase is based on splitting techniques proposed by Juan et al. [16]. In our method, a set of adjacent routes is extracted from the base solution using a probabilistic selection process (line 13). We first select at random the number of routes $k$ to be extracted from our solution, such as $2 \leq k \leq K$ where $K$ is the total number of routes (i.e. vehicles). We use a geometric probability distribution to prioritise the selection of fewer routes, that is to destroy a smaller part of the solution. This probability distribution is controlled by a parameter $\alpha$, such that $0 < \alpha < 1$. This parameter is also used in our `Pack-and-Route` procedure. Starting from a base route, we choose $k-1$ adjacent routes to be extracted from the solution. We apply different geometric criteria to select nearby routes, similar to Juan et al. [16]. Parameters controlling this selection process are adjusted depending on results from previous iterations (lines 7–11). Nodes belonging to selected routes are then removed (line 14) from the solution. This set of customers, together with associated demands, constitutes a sub-instance of the original 2L-VRPB of smaller size, predictably much easier to solve due to the NP-Hard nature of the problem. Sub-problems are also solved by means of the `Pack-and-Route` procedure (line 15). The resulting sub-solution is then merged with non-extracted routes to generate a new solution of the original problem (line 16). If this new solution improves the incumbent one, we accept it as the new base solution (lines 17–20). Finally, the algorithm returns the best found solution when it exhausts its allowed execution time (line 23).

The logic for solving the 2L-VRPB is encoded in the `Pack-and-Route` method. As mentioned, we combine biased-randomised versions of well-known heuristics to tackle routing and packing problems in an integrated manner, contrary to most two-stage approaches for the 2L-VRP.

We start our method by generating an initial dummy solution as described in the savings heuristic by Clarke and Wright [15], i.e. we create a return trip from the depot to each customer using as many vehicles as necessary. Next, we compute the savings associated with each edge and sort them in descending order, as specified in the heuristic. In this process, we omit all edges connecting backhaul to linehaul customers, as sequencing restrictions are imposed in the problem (see Section 2). By ruling out backhaul-linehaul edges, we prevent our algorithm to consider forbidden moves, reducing the number of required feasibility checks. Once the savings list is computed, we rearrange its elements applying a biased-randomised process, so edges associated with higher savings are more likely to be ranked at the top of the list. In our case, a geometric probability distribution, driven by a single parameter $\alpha$ ($0 < \alpha < 1$), is used to induce this skewed behaviour. The biased randomisation of the savings list allows edges to be selected in a different way each time the process is called, while maintaining the logic behind the heuristic [14]. At this point, the algorithm starts an iterative route-merging process. At each iteration, the edge at the top of the savings list is selected. If this edge connects a linehaul customer to a backhaul location, and any or both routes containing these customers is already a mixed linehaul-backhaul route, the edge is discarded. Since all backhaul visits should be done after servicing all linehaul customers in a route, only one such edge can be included per route. Otherwise, both routes are merged yielding a new mixed linehaul-backhaul route. In this case, capacity constraints are not considered for the merging, as by definition the vehicle will be empty after visiting all assigned linehaul customers. If the selected edge is a linehaul-to-linehaul or backhaul-to-backhaul link, routes will be merged subject to: *(i)* there is enough weight capacity in the vehicle to carry all items from both routes; and *(ii)* they can be conveniently loaded, i.e. without overlapping and keeping the sequential order defined by the merged route.

Evaluating packing feasibility might become a time-consuming process. In our approach, we use biased-randomised versions of two well-known effective heuristics: Best-Fit [17] and Touching Perimeter [18]. The Best-Fit heuristic [17] is a constructive and deterministic procedure that dynamically selects the next rectangle (item) to pack inside the truck based on the bottom-left criteria, i.e. among the available items to pack, it always chooses the one that offers the best fit when packed at a free position located at the bottom and on the left. Together with the Best-Fit heuristic, we also use the Touching Perimeter heuristic [18]. Basically, this heuristic works as follows: for each available position, it chooses the rectangle that maximises the associated 'score', i.e. the touching perimeter with other rectangles or with the borders of the truck. In our approach, however, we transform both deterministic procedures into probabilistic ones by introducing a biased-randomisation process similar to the one described in [16], i.e. we use a skewed probability distribution to assign different probabilities to different items, so that the better the fit of a given rectangle the higher its probability of being chosen. In both cases, we use a geometric distribution to skew the search, controlled in the two heuristics by a single parameter $\beta$ ($0 < \beta < 1$). This way, we can run the biased-randomised versions of these heuristics several times, thus increasing our chances of finding a feasible packing solution.

In order to speed up the packing feasibility checking, we use a fast-access memory-based method to determine if we have already computed a packing solution for the same configuration. Since the `Pack-and-Route` procedure is called multiple times during the BR-LNS execution, it is likely that the same packing configuration needs to be checked several times. By keeping track of the best found packing solutions for different configurations, we can significantly reduce the computational burden of our method without losing the solutions quality. If the current configuration is already included in the cache memory, routes are merged. Otherwise, we first use the biased-randomised Best-Fit heuristic to compute a packing solution. If the current configuration is yielded as unfeasible by the Best-Fit method, a biased-randomised Touching Perimeter heuristic is called. The process is repeated a number of times ($maxPackIter$ parameter) before the current configuration is finally disregarded as feasible. Whenever any of the two heuristic methods finds a feasible packing solution, it is stored in the packing cache memory and the process is stopped.

Finally, when a complete solution is computed, we use a fast memory-based technique to possibly improve its associated routes. As in the packing case, we store previously computed routes and packing plans for a given set of customers in a fast-access cache memory. In both cases, we keep in memory the best solution found so far for a specific set of items to be packed or for a specific set of customers to be visited – depending if we consider the memory cache associated with the packing process or the one associated with the routing process. If the obtained solution contains the same set with a higher cost, the route stored in the cache memory is retrieved and the solution is updated. Otherwise, we add (or update) the route to the cache memory for subsequent iterations.

Notice that an important advantage of our approach is its relative simplicity. Our method only uses three parameters, namely $\alpha$, $\beta$, and $maxPackIter$. This significantly reduces the need for fine-tuning processes and its sensitivity to particular problem characteristics, providing a robust method able to perform efficiently across different instances. The values of the parameters were established after a quick tuning process in which different combinations of values were tested for a random sample of instances. After this process, the ‘routing’ biased-randomisation parameter, $\alpha$, was set to 0.3; the ‘packing’ biased-randomisation parameter, $\beta$, was set to a random number in the interval (0.06, 0.23); and, finally, the $maxPackIter$ parameter was set to be equal to the number of items (rectangles) in each instance.

Moreover, the pseudo-random nature of our method makes it suitable for parallelisation, thus potentially providing high-quality results in low computational times.

## 5. Computational Experiments

In order to test the performance of the proposed methodology, we conducted two different experiments. First, we tackle a set of classical 2L-VRP benchmark instances, without backhauls but considering both oriented and non-oriented loading. In order to test the quality of our approach for the 2L-VRP, our results are compared against those obtained using other state-of-the-art metaheuristic approaches. New best solutions are found, both for the $2|SO|L$ and the $2|SR|L$ versions of the 2L-VRP –which constitutes a first noticeable contribution of our work. Then, once the efficiency of our approach has been proved for the 2L-VRP case, we design and solve a new set of instances for the 2L-VRPB –which is a second and even more noteworthy contribution. The instances employed were generated from the classical 2L-VRP benchmark sets to include backhauls. Additionally, we tested our approach over a set of classical benchmark instances for the VRPB [24] to assess the efficiency of the routing component of the algorithm. Similarly, we provide further details on the vehicles' utilisation for some of the generated instances to validate the efficacy of our packing approach.

Our algorithm has been implemented using the Java programming language. All experiments were run in a standard PC with an Intel Core i3 processor at 3.4 GHz and 8 GB RAM. The algorithm was executed on the Netbeans platform for Java over Windows 7.

### *5.1. Computational Results for the 2L-VRP*

In order to evaluate the quality of the solutions obtained with our algorithm, we tested its performance on several 2L-VRP benchmark sets introduced by Iori et al. [7] and Gendreau et al. [40]. Notice that the 2L-VRPB can be seen as an extension of the 2L-VRP, i.e. every 2L-VRP instance may be considered as a especial case of 2L-VRPB where the proportion of linehaul customers is 100%. For this reason, our algorithm does not need to be modified to solve 2L-VRP instances. We tested our BR-LNS approach contemplating two different loading configurations, i.e. sequential oriented loading ($2|SO|L$) and sequential non- oriented loading ($2|SR|L$).

Results for the sequential oriented variant of the 2L-VRP ($2|SO|L$) are summarised in Table 1 for classes 1 to 5. All customers in class 1 have an associated demand of just one item of dimensions $1 \times 1$. Therefore, items rotation has no effect in this class. In this table, we include the best solution found for each instance by means of our BR-LNS approach and its corresponding gap ($\Delta$) to the best-known solution ($BKS$) found in the literature. In all cases, we present the best found solution over 5 executions of our algorithm per instance, allowing a maximum running time of 600 seconds. Further details on the computational times required by our approach to attain the presented results can be found in Tables A.8 to A.12 in Appendix A.

Table 1: Results on the 36 instances of the 2L-VRP – 2|SO|L

| # | Class 1 | | | Class 2 | | | Class 3 | | | Class 4 | | | Class 5 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | BKS | BR-LNS | Δ (%) | BKS | BR-LNS | Δ (%) | BKS | BR-LNS | Δ (%) | BKS | BR-LNS | Δ (%) | BKS | BR-LNS | Δ (%) |
| 1 | 278.73 | 278.73 | 0.00 | 290.84 | 290.84 | 0.00 | 284.52 | 301.88 | 6.10 | 294.25 | 294.25 | 0.00 | 278.73 | 278.73 | 0.00 |
| 2 | 334.96 | 334.96 | 0.00 | 347.73 | 347.73 | 0.00 | 352.16 | 352.16 | 0.00 | 342.00 | 342.00 | 0.00 | 334.96 | 334.96 | 0.00 |
| 3 | 358.40 | 358.40 | 0.00 | 403.93 | 403.93 | 0.00 | 394.72 | 394.72 | 0.00 | 368.56 | 368.56 | 0.00 | 358.40 | 358.40 | 0.00 |
| 4 | 430.88 | 430.88 | 0.00 | 440.94 | 440.94 | 0.00 | 440.68 | 440.68 | 0.00 | 447.37 | 447.37 | 0.00 | 430.88 | 430.88 | 0.00 |
| 5 | 375.28 | 375.28 | 0.00 | 388.72 | 388.72 | 0.00 | 381.69 | 381.69 | 0.00 | 383.87 | 383.87 | 0.00 | 375.28 | 375.28 | 0.00 |
| 6 | 495.85 | 495.85 | 0.00 | 499.08 | 499.08 | 0.00 | 504.68 | 504.68 | 0.00 | 498.32 | 498.32 | 0.00 | 495.85 | 495.85 | 0.00 |
| 7 | 568.56 | 568.56 | 0.00 | 734.65 | 734.65 | 0.00 | 709.72 | 709.72 | 0.00 | 703.49 | 703.49 | 0.00 | 658.64 | 658.64 | 0.00 |
| 8 | 568.56 | 568.56 | 0.00 | 725.91 | 725.91 | 0.00 | 741.12 | 741.12 | 0.00 | 697.92 | 697.92 | 0.00 | 621.85 | 646.46 | 3.96 |
| 9 | 607.65 | 607.65 | 0.00 | 611.49 | 611.49 | 0.00 | 613.90 | 619.48 | 0.91 | 625.10 | 625.13 | 0.00 | 607.65 | 607.65 | 0.00 |
| 10 | 535.74 | 535.80 | 0.01 | 700.20 | 700.20 | 0.00 | 628.93 | 637.46 | 1.36 | 715.82 | 717.83 | 0.28 | 690.96 | 694.71 | 0.54 |
| 11 | 505.01 | 505.01 | 0.00 | 721.54 | 723.34 | 0.25 | 717.37 | 720.63 | 0.45 | 815.68 | **811.56** | -0.51 | 636.77 | 642.20 | 0.85 |
| 12 | 610.00 | 610.00 | 0.00 | 619.63 | 619.63 | 0.00 | 610.00 | 610.00 | 0.00 | 618.23 | 618.23 | 0.00 | 610.23 | 610.23 | 0.00 |
| 13 | 2006.34 | 2006.34 | 0.00 | 2669.39 | 2669.39 | 0.00 | 2486.44 | 2497.42 | 0.44 | 2609.36 | 2622.45 | 0.50 | 2421.88 | 2434.99 | 0.54 |
| 14 | 837.67 | 837.67 | 0.00 | 1101.61 | **1090.55** | -1.00 | 1085.42 | **1069.43** | -1.47 | 983.20 | 988.03 | 0.49 | 924.27 | 943.02 | 2.03 |
| 15 | 837.67 | 837.67 | 0.00 | 1041.75 | 1082.22 | 3.88 | 1181.68 | 1181.68 | 0.00 | 1246.49 | 1246.69 | 0.02 | 1230.40 | 1230.37 | 0.00 |
| 16 | 698.61 | 698.61 | 0.00 | 698.61 | 698.61 | 0.00 | 698.61 | 698.61 | 0.00 | 708.20 | 712.30 | 0.58 | 698.61 | 698.61 | 0.00 |
| 17 | 861.79 | 861.79 | 0.00 | 870.86 | 876.05 | 0.60 | 861.79 | 863.27 | 0.17 | 861.79 | 861.79 | 0.00 | 861.79 | 861.79 | 0.00 |
| 18 | 723.54 | 723.54 | 0.00 | 1053.09 | 1059.74 | 0.63 | 1103.45 | 1115.67 | 1.11 | 1134.11 | 1144.33 | 0.90 | 926.53 | 928.87 | 0.25 |
| 19 | 524.61 | 524.61 | 0.00 | 792.42 | 792.58 | 0.02 | 801.13 | 802.76 | 0.20 | 801.21 | 801.66 | 0.06 | 652.58 | 658.01 | 0.83 |
| 20 | 241.97 | 241.97 | 0.00 | 547.82 | 551.70 | 0.71 | 541.58 | 546.11 | 0.84 | 552.91 | **551.61** | -0.23 | 478.73 | 490.80 | 2.52 |
| 21 | 687.60 | 687.60 | 0.00 | 1060.72 | 1063.31 | 0.24 | 1150.85 | 1157.22 | 0.55 | 1006.21 | 1010.75 | 0.45 | 893.18 | 910.31 | 1.92 |
| 22 | 740.66 | 740.66 | 0.00 | 1081.44 | 1081.45 | 0.00 | 1094.66 | 1099.97 | 0.49 | 1089.27 | 1097.58 | 0.76 | 948.60 | 959.18 | 1.12 |
| 23 | 835.26 | 835.26 | 0.00 | 1093.27 | 1112.93 | 1.80 | 1117.54 | 1120.41 | 0.26 | 1093.01 | 1098.23 | 0.48 | 950.25 | 964.16 | 1.46 |
| 24 | 1024.69 | 1024.69 | 0.00 | 1222.43 | 1226.48 | 0.33 | 1118.44 | 1120.79 | 0.21 | 1141.97 | 1146.71 | 0.41 | 1048.69 | 1049.76 | 0.10 |
| 25 | 826.14 | 826.14 | 0.00 | 1458.83 | **1456.13** | -0.19 | 1436.57 | 1440.10 | 0.25 | 1435.18 | 1442.97 | 0.54 | 1183.63 | 1202.25 | 1.57 |
| 26 | 819.56 | 819.56 | 0.00 | 1327.47 | 1333.02 | 0.42 | 1396.52 | 1405.54 | 0.65 | 1447.03 | 1449.64 | 0.18 | 1252.65 | 1261.07 | 0.67 |
| 27 | 1082.65 | 1082.65 | 0.00 | 1367.85 | 1375.90 | 0.59 | 1423.74 | 1425.09 | 0.09 | 1357.75 | 1362.70 | 0.36 | 1270.34 | 1294.92 | 1.94 |
| 28 | 1040.70 | 1042.12 | 0.14 | 2699.21 | **2683.66** | -0.58 | 2787.24 | 2808.47 | 0.76 | 2700.66 | 2710.67 | 0.37 | 2399.25 | 2423.74 | 1.02 |
| 29 | 1162.96 | 1162.96 | 0.00 | 2289.84 | 2317.04 | 1.19 | 2172.69 | 2210.53 | 1.74 | 2312.37 | 2316.97 | 0.20 | 2191.69 | 2197.37 | 0.26 |
| 30 | 1028.42 | 1028.42 | 0.00 | 1875.38 | 1888.71 | 0.71 | 1915.42 | 1923.31 | 0.41 | 1910.54 | 1921.09 | 0.55 | 1575.64 | 1597.44 | 1.38 |
| 31 | 1299.56 | **1299.21** | -0.03 | 2369.07 | **2353.87** | -0.64 | 2360.63 | 2376.74 | 0.68 | 2469.40 | 2491.17 | 0.88 | 2072.19 | 2073.95 | 0.08 |
| 32 | 1296.91 | **1296.18** | -0.06 | 2384.29 | **2370.94** | -0.56 | 2325.74 | 2329.44 | 0.16 | 2357.57 | **2353.05** | -0.19 | 2031.92 | 2055.56 | 1.16 |
| 33 | 1298.02 | **1297.50** | -0.04 | 2376.58 | **2373.81** | -0.12 | 2469.85 | **2465.77** | -0.17 | 2470.76 | **2464.76** | -0.24 | 2054.29 | 2073.33 | 0.93 |
| 34 | 708.39 | 709.08 | 0.10 | 1226.98 | 1230.43 | 0.28 | 1253.88 | 1263.81 | 0.79 | 1242.26 | 1248.65 | 0.51 | 1062.18 | 1082.77 | 1.94 |
| 35 | 865.39 | **864.63** | -0.09 | 1447.30 | **1441.68** | -0.39 | 1529.77 | **1529.10** | -0.04 | 1558.69 | 1567.09 | 0.54 | 1281.90 | 1313.31 | 2.45 |
| 36 | 585.46 | 590.16 | 0.80 | 1784.57 | **1768.34** | -0.91 | 1869.38 | **1862.13** | -0.39 | 1740.64 | 1742.93 | 0.13 | 1549.51 | 1567.06 | 1.13 |
| *Avg.* | 769.56 | 769.69 | 0.02 | 1175.71 | 1177.36 | 0.20 | 1182.29 | 1186.88 | 0.46 | 1187.26 | 1190.62 | 0.22 | 1057.25 | 1066.85 | 0.85 |

Additionally, we compare our figures to results obtained by other state-of-the-art heuristics for the 2L-VRP. In particular, we measure the performance of our algorithm with the following works, whose execution parameters are also specified next:

(a) Fuellerer et al. [8]: they use and Ant Colony Optimisation (ACO) approach, coded in C++. The authors ran the experiments in an Intel Pentium 4 at 3.2 GHz on a Linux operating system. They present the best solution obtained over 10 executions of their algorithm for each instance. The maximum allowed time per instance and execution is 3 hours.

(b) Zachariadis et al. [46]: they propose a metaheuristic called Promise Routing-Memory Packing (PRMP), coded in Visual C#. The experiments were run in a single-core Intel E6600 at 2.4 GHz. In this case, the authors also provide the best obtained solution over 10 executions of their algorithm per instance, allowing execution times longer than 3 hours for some of the experiments.

(c) Wei et al. [47]: the authors presented a Variable Neighbourhood Search (VNS) approach, coded in C++. They used a computer with a quad-core processor Intel Xeon E5430 at 2.6 GHz and 8 GB RAM running a CentOS 5 Linux operating system. They provide the best found solution over 10 executions of their algorithm per instance. The execution time is adjusted according to the instance scale: 900 seconds for $n \leq 50$, 1800 seconds for $50 < n \leq 100$, and 3600 seconds for $n > 100$, where $n$ is the number of customers.

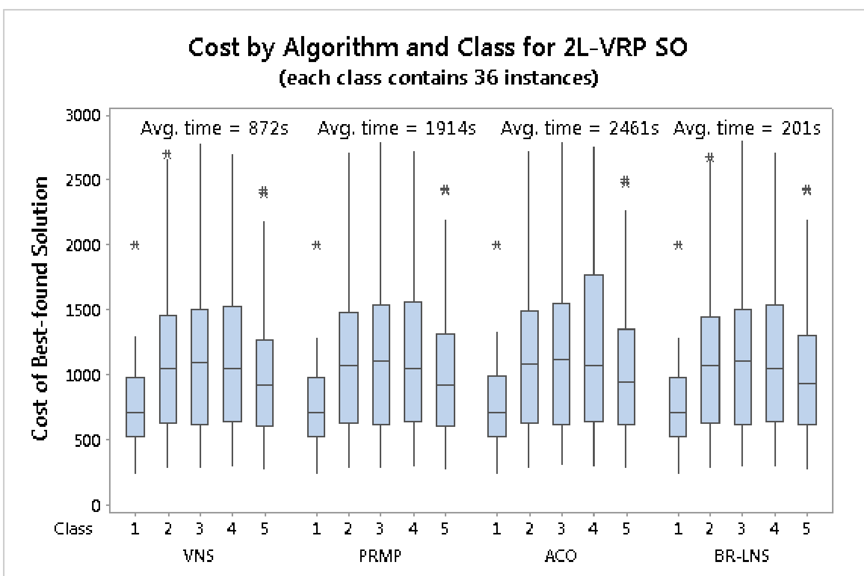


Figure 2: Comparison of metaheuristics for the 2L-VRP without items rotation.

We observe that our BR-LNS algorithm has produced 20 new best solutions (in bold) for several instances of the $2|SO|L$ variant of the 2L-VRP. Also, for the remaining instances our approach is able to match the best-known solution or yield a result with a very small gap. As shown in Figure 2, in terms of the quality of the obtained solutions our algorithm is perfectly comparable to other state-of-the-art approaches. Additionally, it employs much lower computational times than any other approach in the comparison (average times, in seconds, for each instance and class: VNS = 872, PRMP = 1914, ACO = 2461, BR-LNS = 201). Detailed results of this comparison can be found in Tables A.8, A.9, A.10, A.11, and A.12 in Appendix A.

Regarding the 2L-VRP with sequential non-oriented loading ($2|SR|L$), the only previous results found in the literature belong to Fuellerer et al. [8]. To the best of our knowledge, they are the first and only authors to deal with all four variants of the 2L-VRP. We provide our results for the $2|SR|L$ configuration in Table 2 for classes 2 to 5. Notice that, since class 1 has unitary square items, its results for the $2|SR|L$ will be the same as the ones already provided for the $2|SO|L$. We compare our results to those obtained by Fuellerer et al. [8] by means of their ACO approach. As a reference, we also include the gap between both solutions ($\Delta$) for each instance. For the two approaches, execution parameters are as previously described, i.e. 10 executions per instance and 3 hours time limit for Fuellerer et al. [8] approach and 5 executions per instance and 600 seconds for our BR-LNS algorithm. For the interested reader, detailed results for this comparison including execution times are provided in Table A.13 in Appendix A. Notice that solution costs are generally lower for the same instance when items rotation is considered (compared to oriented loading; c.f. Table 1). This suggests that allowing items rotation could yield noticeable savings in real-life applications.

As shown in Table 2, our approach produced a significant number of best solutions (in bold) for many instances across all classes. Concretely, we found 103 new best-known solutions for a total of 144 tested instances. If the solution is not improved, we are generally able to match results by Fuellerer et al. [8] or remain very close (gap for all instances but one is below 1%). As it can be seen in Figure 3, our algorithm offers better overall performance, both in terms of solution quality as in terms of computing times (average times, in seconds, for each class and instance: ACO = 2247, BR-LNS = 214).

Table 2: Results on the 36 instances of the 2L-VRP – 2|SR|L

| # | Class 2 | | | Class 3 | | | Class 4 | | | Class 5 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | ACO | BR-LNS | Δ (%) | ACO | BR-LNS | Δ (%) | ACO | BR-LNS | Δ (%) | ACO | BR-LNS | Δ (%) |
| 1 | 278.73 | 278.73 | 0.00 | 284.52 | **284.23** | -0.10 | 282.95 | 282.95 | 0.00 | 280.60 | **278.73** | -0.67 |
| 2 | 334.96 | 334.96 | 0.00 | 352.16 | 352.16 | 0.00 | 342.00 | **334.96** | -2.06 | 334.96 | 334.96 | 0.00 |
| 3 | 384.93 | 384.93 | 0.00 | 394.72 | **390.55** | -1.06 | 368.56 | **362.41** | -1.67 | 358.40 | 358.40 | 0.00 |
| 4 | 430.88 | 430.88 | 0.00 | 430.88 | 430.88 | 0.00 | 447.37 | 447.37 | 0.00 | 430.88 | 430.88 | 0.00 |
| 5 | 375.28 | 375.28 | 0.00 | 379.94 | 379.94 | 0.00 | 383.88 | 383.87 | 0.00 | 375.28 | 375.28 | 0.00 |
| 6 | 498.16 | **495.85** | -0.46 | 498.16 | 498.16 | 0.00 | 498.32 | 498.32 | 0.00 | 495.85 | 495.85 | 0.00 |
| 7 | 716.82 | 716.82 | 0.00 | 706.99 | **678.75** | -3.99 | 702.45 | **686.26** | -2.31 | 658.64 | 658.64 | 0.00 |
| 8 | 674.20 | **671.75** | -0.36 | 741.12 | 741.12 | 0.00 | 705.89 | **692.47** | -1.90 | 639.18 | 640.01 | 0.13 |
| 9 | 607.65 | 607.65 | 0.00 | 607.65 | 607.65 | 0.00 | 625.13 | **625.10** | -0.01 | 607.65 | 607.65 | 0.00 |
| 10 | 685.21 | **684.37** | -0.12 | 617.62 | **615.68** | -0.31 | 722.70 | **708.68** | -1.94 | 693.15 | **691.04** | -0.30 |
| 11 | 694.60 | 702.74 | 1.17 | 706.73 | 711.80 | 0.72 | 800.88 | **777.04** | -2.98 | 644.46 | 644.93 | 0.07 |
| 12 | 615.87 | **610.00** | -0.95 | 610.23 | **610.00** | -0.04 | 619.21 | **614.23** | -0.80 | 610.23 | 610.23 | 0.00 |
| 13 | 2526.07 | 2534.97 | 0.35 | 2469.98 | **2436.06** | -1.37 | 2623.65 | **2605.34** | -0.70 | 2434.99 | **2386.96** | -1.97 |
| 14 | 1041.61 | **1032.01** | -0.92 | 1012.46 | **1009.46** | -0.30 | 988.25 | **982.16** | -0.62 | 927.79 | **922.02** | -0.62 |
| 15 | 1009.87 | 1009.87 | 0.00 | 1170.82 | **1142.18** | -2.45 | 1245.94 | **1171.41** | -5.98 | 1234.87 | **1204.30** | -2.48 |
| 16 | 698.61 | 698.61 | 0.00 | 698.61 | 698.61 | 0.00 | 703.35 | 703.35 | 0.00 | 698.61 | 698.61 | 0.00 |
| 17 | 861.79 | 861.79 | 0.00 | 861.79 | 861.79 | 0.00 | 861.79 | 861.79 | 0.00 | 861.79 | 861.79 | 0.00 |
| 18 | 989.21 | **988.36** | -0.09 | 1031.94 | **1030.69** | -0.12 | 1128.25 | **1104.08** | -2.14 | 926.39 | **925.72** | -0.07 |
| 19 | 732.64 | **731.93** | -0.10 | 757.59 | 764.14 | 0.87 | 796.42 | **772.39** | -3.02 | 664.28 | **653.92** | -1.56 |
| 20 | 496.93 | 500.98 | 0.81 | 536.58 | **519.15** | -3.25 | 549.38 | **542.90** | -1.18 | 488.28 | **478.37** | -2.03 |
| 21 | 998.48 | **988.25** | -1.02 | 1126.49 | **1102.63** | -2.12 | 1006.61 | **976.38** | -3.00 | 911.01 | **892.18** | -2.07 |
| 22 | 1009.25 | **1000.79** | -0.84 | 1060.79 | **1030.00** | -2.90 | 1089.59 | **1062.94** | -2.45 | 956.71 | **946.90** | -1.03 |
| 23 | 1019.73 | **997.58** | -2.17 | 1090.74 | **1069.58** | -1.94 | 1098.82 | **1077.98** | -1.90 | 959.79 | **949.49** | -1.07 |
| 24 | 1183.02 | **1174.29** | -0.74 | 1091.54 | **1076.30** | -1.40 | 1129.79 | **1101.83** | -2.47 | 1059.06 | **1047.80** | -1.06 |
| 25 | 1383.57 | **1373.52** | -0.73 | 1374.51 | **1363.21** | -0.82 | 1442.71 | **1404.84** | -2.62 | 1204.57 | **1183.77** | -1.73 |
| 26 | 1283.32 | **1275.83** | -0.58 | 1372.46 | **1341.19** | -2.28 | 1450.79 | **1406.40** | -3.06 | 1260.75 | **1254.96** | -0.46 |
| 27 | 1310.13 | **1295.03** | -1.15 | 1399.26 | **1369.42** | -2.13 | 1369.55 | **1317.17** | -3.82 | 1297.83 | **1259.00** | -2.99 |
| 28 | 2548.28 | **2528.53** | -0.78 | 2710.90 | **2583.70** | -4.69 | 2724.74 | **2655.25** | -2.55 | 2416.88 | **2358.10** | -2.43 |
| 29 | 2197.20 | **2168.94** | -1.29 | 2138.30 | **2084.66** | -2.51 | 2327.21 | **2274.96** | -2.25 | 2206.55 | **2165.02** | -1.88 |
| 30 | 1805.65 | **1770.80** | -1.93 | 1874.66 | **1817.72** | -3.04 | 1908.00 | **1840.99** | -3.51 | 1593.81 | **1558.68** | -2.20 |
| 31 | 2265.21 | **2223.03** | -1.86 | 2326.58 | **2264.96** | -2.65 | 2486.10 | **2392.25** | -3.77 | 2097.62 | **2040.29** | -2.73 |
| 32 | 2258.99 | **2229.46** | -1.31 | 2305.18 | **2235.42** | -3.03 | 2372.77 | **2282.58** | -3.80 | 2062.45 | **2004.86** | -2.79 |
| 33 | 2251.88 | **2208.80** | -1.91 | 2407.87 | **2343.34** | -2.68 | 2479.42 | **2389.32** | -3.63 | 2081.18 | **2018.52** | -3.01 |
| 34 | 1172.76 | **1157.28** | -1.32 | 1238.94 | **1193.34** | -3.68 | 1251.31 | **1211.21** | -3.20 | 1088.21 | **1055.12** | -3.04 |
| 35 | 1375.14 | **1362.09** | -0.95 | 1483.13 | **1444.20** | -2.62 | 1606.54 | **1511.27** | -5.93 | 1314.60 | **1285.15** | -2.24 |
| 36 | 1737.64 | **1680.64** | -3.28 | 1842.36 | **1769.88** | -3.93 | 1765.72 | **1673.25** | -5.24 | 1594.14 | **1527.63** | -4.17 |
| *Avg.* | 1123.73 | 1113.54 | -0.63 | 1158.73 | 1134.79 | -1.50 | 1191.83 | 1159.33 | -2.24 | 1068.65 | 1050.16 | -1.23 |

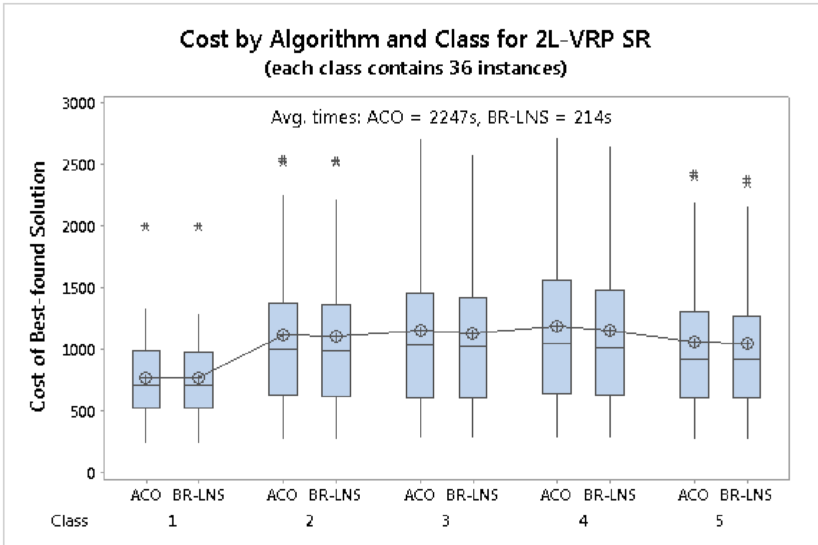


Figure 3: Comparison of metaheuristics for the 2L-VRP with items rotation.

### *5.2. Computational Results for the 2L-VRPB*

Using the method described by Toth and Vigo [24] to generate VRPB instances from classic Euclidean VRP ones, the 2L-VRP instances have been extended to generate new instances for the 2L-VRPB. Thus, we have generated three new 2L-VRPB instances for each 2L-VRP one. These new instances contain 50%, 60%, and 80% linehaul customers. To obtain such linehaul-backhaul distributions, we select a customer every two, three, or five customers, respectively, to be a backhaul location. These linehaul/backhaul configurations are represented by 1/1, 2/1, and 4/1, respectively. This way, we generated a total of 540 2L-VRPB instances derived from the 180 2L-VRP instances contained in the original sets.

Combined with the sequential loading constraint, we also consider two different loading configurations for each instance, i.e. oriented and non-oriented. Hence, we obtain two results per instance: *(i)* considering sequential oriented loading ($2|SO|L$); and *(ii)* with sequential loading allowing items rotation ($2|SR|L$).

Table 3 presents results for instances with a 1/1 linehaul/backhaul configuration. Since all customers in class 1 have an associated demand of just one item of dimensions $1 \times 1$, items rotation has no efect on the solution quality. Thus, the same result is valid for both loading configurations. Results for instances with 2/1 and 4/1 linehaul/backhaul configurations are included in Tables 4 and 5, respectively. Further details on the required computational times and the number of vehicles ($K$) used in each solution can be found in Tables A.14 to A.18 in Appendix A.

Table 3: Results on the 36 instances of the 2L-VRPB with a linehaul/backhaul ratio of 1/1, for each class, and considering oriented loading (2|SO|L) and items rotation (2|SR|L).

| Linehaul / Backhaul : 1 / 1 | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| # | Class 1 | Class 2 | | Class 3 | | Class 4 | | Class 5 | |
| | | 2\|SO\|L | 2\|SR\|L | 2\|SO\|L | 2\|SR\|L | 2\|SO\|L | 2\|SR\|L | 2\|SO\|L | 2\|SR\|L |
| 1 | 301.99 | 308.76 | 307.63 | 308.76 | 307.63 | 312.12 | 312.12 | 307.63 | 301.99 |
| 2 | 308.76 | 308.76 | 308.76 | 308.76 | 308.76 | 308.76 | 308.76 | 308.76 | 308.76 |
| 3 | 335.54 | 336.40 | 335.54 | 345.66 | 344.11 | 335.54 | 335.54 | 335.54 | 335.54 |
| 4 | 375.12 | 375.12 | 375.12 | 375.12 | 375.12 | 375.12 | 375.12 | 375.12 | 375.12 |
| 5 | 372.12 | 376.84 | 372.12 | 373.71 | 373.71 | 372.12 | 372.12 | 372.12 | 372.12 |
| 6 | 432.30 | 428.88 | 428.88 | 432.30 | 432.30 | 432.30 | 432.30 | 432.30 | 432.30 |
| 7 | 689.32 | 692.26 | 689.32 | 691.85 | 691.85 | 699.27 | 699.27 | 689.32 | 689.32 |
| 8 | 689.32 | 698.87 | 689.32 | 718.89 | 718.89 | 692.26 | 692.26 | 677.52 | 677.52 |
| 9 | 494.03* | 501.48 | 494.03 | 494.03 | 494.03 | 500.57 | 500.57 | 494.03 | 494.03 |
| 10 | 502.77 | 610.45 | 556.61 | 536.29 | 521.00 | 589.43 | 576.93 | 571.68 | 568.53 |
| 11 | 502.77 | 603.37 | 573.79 | 581.42 | 562.99 | 644.27 | 626.79 | 573.31 | 571.68 |
| 12 | 471.46 | 482.63 | 476.08 | 471.46 | 471.46 | 475.76 | 475.76 | 471.46 | 471.46 |
| 13 | 2276.57 | 2399.98 | 2360.14 | 2384.40 | 2342.09 | 2354.57 | 2354.57 | 2326.80 | 2326.80 |
| 14 | 751.69 | 870.04 | 783.87 | 878.23 | 784.59 | 777.60 | 767.48 | 771.31 | 769.21 |
| 15 | 751.69 | 850.73 | 786.21 | 853.62 | 767.08 | 909.02 | 895.65 | 907.13 | 901.79 |
| 16 | 543.09 | 549.86 | 542.60 | 544.24 | 544.24 | 543.39 | 542.60 | 542.60 | 542.60 |
| 17 | 638.14 | 635.94 | 635.94 | 635.94 | 635.94 | 638.14 | 635.94 | 635.94 | 635.94 |
| 18 | 834.86 | 937.03 | 898.99 | 919.65 | 865.67 | 918.57 | 899.17 | 845.35 | 845.35 |
| 19 | 562.83 | 655.44 | 616.69 | 655.97 | 639.40 | 637.33 | 633.98 | 617.50 | 604.80 |
| 20 | 319.72 | 419.92 | 372.83 | 397.05 | 383.65 | 398.26 | 385.51 | 375.20 | 372.24 |
| 21 | 721.78 | 876.37 | 801.36 | 892.75 | 842.31 | 844.89 | 824.78 | 783.33 | 774.43 |
| 22 | 721.68 | 872.10 | 822.11 | 862.65 | 841.57 | 899.02 | 853.02 | 805.17 | 792.44 |
| 23 | 746.90 | 880.09 | 832.02 | 860.55 | 837.07 | 862.22 | 830.68 | 802.86 | 798.15 |
| 24 | 838.96 | 920.51 | 889.85 | 890.40 | 850.98 | 896.58 | 868.43 | 844.15 | 834.08 |
| 25 | 889.59 | 1144.05 | 1060.56 | 1102.54 | 1051.22 | 1091.96 | 1068.36 | 984.61 | 983.36 |
| 26 | 779.21 | 1031.22 | 956.30 | 1039.09 | 953.12 | 1096.63 | 1033.00 | 903.86 | 899.37 |
| 27 | 964.88 | 1073.48 | 1026.46 | 1089.58 | 1055.65 | 1058.67 | 1013.58 | 1011.70 | 1009.33 |
| 28 | 1022.91 | 1780.33 | 1671.31 | 1801.48 | 1690.30 | 1813.13 | 1751.68 | 1616.89 | 1595.35 |
| 29 | 1217.36* | 1727.00 | 1572.62 | 1638.68 | 1539.33 | 1667.36 | 1634.34 | 1625.58 | 1625.08 |
| 30 | 1050.11 | 1415.14 | 1312.54 | 1396.25 | 1339.90 | 1385.71 | 1341.79 | 1236.57 | 1209.27 |
| 31 | 1216.24 | 1686.66 | 1589.06 | 1698.68 | 1614.75 | 1730.54 | 1677.90 | 1545.89 | 1513.71 |
| 32 | 1202.83 | 1700.82 | 1584.83 | 1679.53 | 1608.71 | 1687.62 | 1623.55 | 1521.70 | 1503.93 |
| 33 | 1213.71 | 1716.05 | 1609.70 | 1715.24 | 1652.67 | 1732.86 | 1663.38 | 1505.30 | 1496.67 |
| 34 | 702.84 | 890.10 | 825.33 | 908.90 | 868.20 | 877.18 | 855.08 | 808.02 | 794.92 |
| 35 | 747.01 | 1006.72 | 897.77 | 1020.11 | 956.36 | 1027.38 | 982.16 | 893.69 | 876.87 |
| 36 | 488.96 | 1090.58 | 1016.32 | 1126.35 | 1046.33 | 1052.64 | 997.65 | 946.13 | 926.31 |

* In these instances, linehaul and backhaul customer set were swapped, to avoid $K_L < K_B$.

Similarly to the 2L-VRP instances, notice that the solutions obtained allowing items rotation are slightly better than those obtained with sequential loading. Again, this suggests that considering items rotation may yield significant savings in real-life applications. Likewise, notice that the cost of some instances tends to increase as configurations with more linehaul customers are considered.

As mentioned in Section 3, only Malapert et al. [12] have previously analysed

Table 4: Results on the 36 instances of the 2L-VRPB with a linehaul/backhaul ratio of 2/1, for each class, and considering oriented loading (2|SO|L) and items rotation (2|SR|L).

| Linehaul / Backhaul : 2 / 1 | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| # | Class 1 | Class 2 | | Class 3 | | Class 4 | | Class 5 | |
| | | 2\|SO\|L | 2\|SR\|L | 2\|SO\|L | 2\|SR\|L | 2\|SO\|L | 2\|SR\|L | 2\|SO\|L | 2\|SR\|L |
| 1 | 274.25 | 274.25 | 274.25 | 274.25 | 274.25 | 274.25 | 274.25 | 284.22 | 274.25 |
| 2 | 323.52 | 323.52 | 323.52 | 323.52 | 323.52 | 323.52 | 323.52 | 323.52 | 323.52 |
| 3 | 352.70 | 380.89 | 358.42 | 355.02 | 352.70 | 352.83 | 352.83 | 352.70 | 352.70 |
| 4 | 396.11 | 397.66 | 397.66 | 396.11 | 396.11 | 396.11 | 396.11 | 396.11 | 396.11 |
| 5 | 365.55 | 378.70 | 365.55 | 365.55 | 365.55 | 373.66 | 370.07 | 365.55 | 365.55 |
| 6 | 405.99 | 408.53 | 405.99 | 405.99 | 405.99 | 425.35 | 421.56 | 405.99 | 405.99 |
| 7 | 678.88 | 703.67 | 703.67 | 693.58 | 693.46 | 703.67 | 703.19 | 693.58 | 693.58 |
| 8 | 692.49 | 693.58 | 693.58 | 703.67 | 703.67 | 703.67 | 703.67 | 693.58 | 693.58 |
| 9 | 526.48 | 531.24 | 526.48 | 530.35 | 526.48 | 526.48 | 526.48 | 526.48 | 526.48 |
| 10 | 550.62 | 611.94 | 593.98 | 576.50 | 554.57 | 637.45 | 637.45 | 590.46 | 590.46 |
| 11 | 550.62 | 626.35 | 611.00 | 565.46 | 560.43 | 638.45 | 612.41 | 565.82 | 563.95 |
| 12 | 497.63 | 515.80 | 505.52 | 497.63 | 497.63 | 498.25 | 498.25 | 497.63 | 497.63 |
| 13 | 2184.46 | 2407.68 | 2345.91 | 2239.93 | 2208.95 | 2478.79 | 2439.28 | 2329.21 | 2300.45 |
| 14 | 725.95 | 894.55 | 877.66 | 886.27 | 857.36 | 871.59 | 838.18 | 767.04 | 756.84 |
| 15 | 725.95 | 834.07 | 761.96 | 886.93 | 840.85 | 899.68 | 887.81 | 892.28 | 884.92 |
| 16 | 582.64 | 578.20 | 578.20 | 578.20 | 578.20 | 599.82 | 582.21 | 578.20 | 578.20 |
| 17 | 697.42 | 681.87 | 680.30 | 680.30 | 680.30 | 680.30 | 680.30 | 680.30 | 680.30 |
| 18 | 814.27 | 956.22 | 905.48 | 920.45 | 888.28 | 963.98 | 940.67 | 880.82 | 880.82 |
| 19 | 578.70 | 694.60 | 647.53 | 712.12 | 672.78 | 678.47 | 661.86 | 625.08 | 617.53 |
| 20 | 304.45 | 427.23 | 388.27 | 424.19 | 414.19 | 467.63 | 446.46 | 392.19 | 390.85 |
| 21 | 715.42 | 927.84 | 870.89 | 973.94 | 922.14 | 860.51 | 843.69 | 815.47 | 794.37 |
| 22 | 742.14 | 883.66 | 853.83 | 935.45 | 897.24 | 875.77 | 863.08 | 843.17 | 838.64 |
| 23 | 773.12 | 931.68 | 850.40 | 955.00 | 906.30 | 913.78 | 895.69 | 841.20 | 819.20 |
| 24 | 873.83 | 1007.87 | 963.74 | 948.72 | 912.60 | 948.54 | 917.92 | 884.62 | 881.49 |
| 25 | 830.07 | 1219.02 | 1136.68 | 1141.33 | 1098.11 | 1178.43 | 1129.62 | 1021.60 | 1004.54 |
| 26 | 773.24 | 1095.65 | 1051.25 | 1086.15 | 1024.31 | 1107.94 | 1075.59 | 952.00 | 923.19 |
| 27 | 974.54 | 1173.29 | 1108.04 | 1180.91 | 1116.42 | 1091.93 | 1071.70 | 1074.09 | 1050.65 |
| 28 | 1039.50 | 1925.05 | 1822.48 | 2029.48 | 1878.15 | 1974.14 | 1914.67 | 1827.23 | 1774.85 |
| 29 | 1342.38 | 1846.64 | 1765.09 | 1722.74 | 1661.51 | 1881.43 | 1840.32 | 1798.12 | 1764.25 |
| 30 | 1059.28 | 1560.45 | 1453.13 | 1554.94 | 1470.26 | 1566.45 | 1504.83 | 1326.03 | 1284.32 |
| 31 | 1278.37 | 1904.79 | 1785.43 | 1866.44 | 1781.28 | 1934.79 | 1880.64 | 1719.53 | 1688.12 |
| 32 | 1291.09 | 1894.47 | 1778.63 | 1846.20 | 1777.99 | 1875.74 | 1815.40 | 1662.50 | 1624.16 |
| 33 | 1305.80 | 1905.83 | 1764.91 | 1947.88 | 1847.64 | 1950.56 | 1880.26 | 1663.11 | 1628.61 |
| 34 | 633.81 | 953.01 | 898.38 | 988.73 | 924.61 | 953.31 | 921.45 | 867.07 | 849.22 |
| 35 | 793.11 | 1133.30 | 1054.42 | 1156.39 | 1094.68 | 1205.36 | 1181.60 | 1026.55 | 1011.02 |
| 36 | 550.55 | 1267.03 | 1204.45 | 1315.68 | 1238.56 | 1230.27 | 1182.88 | 1109.22 | 1087.75 |

the 2L-VRPB. In their work, they present a Constraint Programming model based on a scheduling approach to tackle the problem considering sequential loading and oriented configurations. However, the authors do not provide any results obtained with their model. As far as we are concerned, we present the first results obtained for 2L-VRPB benchmark instances, with and without items rotation. Therefore, we cannot compare them against other state-of-the-art approaches for the problem. However, we may assess the routing and packing components of our algorithm alone to ensure the validity of our approach.

Table 5: Results on the 36 instances of the 2L-VRPB with a linehaul/backhaul ratio of 4/1, for each class, and considering oriented loading (2|SO|L) and items rotation (2|SR|L).

| Linehaul / Backhaul : 4 / 1 | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| # | Class 1 | Class 2 | | Class 3 | | Class 4 | | Class 5 | |
| | | 2\|SO\|L | 2\|SR\|L | 2\|SO\|L | 2\|SR\|L | 2\|SO\|L | 2\|SR\|L | 2\|SO\|L | 2\|SR\|L |
| 1 | 259.97 | 259.97 | 259.97 | 260.22 | 259.97 | 275.25 | 259.97 | 259.97 | 259.97 |
| 2 | 299.64 | 314.14 | 299.64 | 322.42 | 310.97 | 299.64 | 299.64 | 299.64 | 299.64 |
| 3 | 349.12 | 350.83 | 349.12 | 367.86 | 356.68 | 356.76 | 356.76 | 349.12 | 349.12 |
| 4 | 415.83 | 395.42 | 395.42 | 395.42 | 395.42 | 410.20 | 410.20 | 395.42 | 395.42 |
| 5 | 376.68 | 376.68 | 376.68 | 376.68 | 376.68 | 385.74 | 385.74 | 376.68 | 376.68 |
| 6 | 432.83 | 432.85 | 432.83 | 432.83 | 432.83 | 432.83 | 432.83 | 432.83 | 432.83 |
| 7 | 598.68 | 723.39 | 694.63 | 674.70 | 654.42 | 674.28 | 631.28 | 631.28 | 631.28 |
| 8 | 598.68 | 683.64 | 663.45 | 713.49 | 675.93 | 660.95 | 654.42 | 603.43 | 600.65 |
| 9 | 571.75 | 573.06 | 571.75 | 573.06 | 571.75 | 571.75 | 571.75 | 571.75 | 571.75 |
| 10 | 512.06 | 642.58 | 613.00 | 613.95 | 613.95 | 663.73 | 657.26 | 609.63 | 602.65 |
| 11 | 512.06 | 662.43 | 632.01 | 663.37 | 658.48 | 737.89 | 696.65 | 614.38 | 612.30 |
| 12 | 523.41 | 546.33 | 532.12 | 524.53 | 522.56 | 534.87 | 522.56 | 522.56 | 522.56 |
| 13 | 1997.84 | 2489.25 | 2324.70 | 2468.80 | 2294.43 | 2518.66 | 2462.42 | 2286.38 | 2251.06 |
| 14 | 746.28 | 1017.55 | 902.70 | 879.84 | 868.60 | 900.65 | 888.48 | 863.12 | 863.12 |
| 15 | 746.28 | 963.49 | 887.45 | 1024.84 | 998.30 | 1085.14 | 1011.12 | 1002.07 | 1000.43 |
| 16 | 613.19 | 614.67 | 609.85 | 610.99 | 610.99 | 622.18 | 609.85 | 610.99 | 609.85 |
| 17 | 725.83 | 734.15 | 727.90 | 723.17 | 722.62 | 724.47 | 723.17 | 722.62 | 722.62 |
| 18 | 791.40 | 1000.84 | 947.31 | 971.94 | 965.96 | 989.86 | 982.23 | 909.63 | 896.74 |
| 19 | 567.89 | 698.50 | 669.58 | 742.96 | 714.81 | 722.10 | 710.00 | 637.06 | 636.34 |
| 20 | 288.90 | 460.16 | 435.36 | 466.63 | 448.55 | 500.80 | 489.69 | 445.67 | 435.31 |
| 21 | 703.81 | 965.26 | 900.67 | 1035.99 | 970.50 | 906.63 | 878.72 | 848.91 | 828.99 |
| 22 | 733.42 | 990.59 | 936.02 | 968.10 | 938.27 | 996.26 | 976.26 | 888.16 | 880.79 |
| 23 | 794.85 | 958.56 | 886.38 | 986.13 | 945.58 | 956.42 | 933.62 | 873.26 | 866.27 |
| 24 | 904.53 | 1061.97 | 1022.57 | 999.72 | 953.55 | 1002.35 | 980.42 | 922.58 | 912.80 |
| 25 | 859.97 | 1312.22 | 1227.70 | 1255.11 | 1179.71 | 1271.21 | 1240.41 | 1088.57 | 1079.72 |
| 26 | 833.59 | 1259.05 | 1173.86 | 1229.65 | 1179.77 | 1283.55 | 1259.88 | 1125.75 | 1099.40 |
| 27 | 1004.20 | 1245.97 | 1164.18 | 1283.22 | 1226.13 | 1207.54 | 1172.75 | 1149.52 | 1138.12 |
| 28 | 1059.68 | 2303.88 | 2139.68 | 2334.18 | 2192.05 | 2186.90 | 2103.46 | 2047.11 | 2007.87 |
| 29 | 1210.77 | 2009.95 | 1935.64 | 1986.48 | 1878.23 | 1905.98 | 1854.41 | 1935.20 | 1881.82 |
| 30 | 1067.26 | 1687.05 | 1585.44 | 1663.83 | 1611.83 | 1642.08 | 1570.02 | 1445.86 | 1406.47 |
| 31 | 1260.15 | 2048.22 | 1955.70 | 2029.25 | 1934.67 | 2141.65 | 2042.84 | 1864.63 | 1812.71 |
| 32 | 1260.15 | 2038.96 | 1877.14 | 2026.01 | 1926.61 | 2005.18 | 1940.05 | 1795.95 | 1757.09 |
| 33 | 1295.28 | 2081.76 | 1947.88 | 2191.51 | 2058.68 | 2147.33 | 2053.65 | 1880.20 | 1836.32 |
| 34 | 654.71 | 1066.72 | 1003.07 | 1087.44 | 1031.75 | 1078.34 | 1047.32 | 946.10 | 932.51 |
| 35 | 839.02 | 1253.38 | 1160.50 | 1308.31 | 1212.11 | 1318.93 | 1259.19 | 1146.00 | 1115.01 |
| 36 | 584.62 | 1500.73 | 1401.12 | 1494.50 | 1422.05 | 1448.06 | 1376.93 | 1309.89 | 1281.93 |

In order to test the efficiency of the routing component of our algorithm, we have used it to solve a set of classical VRPB benchmark instances proposed by Toth and Vigo [24]. For each instance, our algorithm was run 5 times, using different seeds. Each run employed a maximum time of 100 seconds. Table 6 summarises these computational experiments. For each instance, we provide its characteristics: number of nodes ($N$), number of linehaul customers ($n$), number of backhaul customers ($m$), capacity of each vehicle ($Q$), and number of vehicles ($K$). We compare our solutions ($BR-LNS$) to the best known ones

($BKS$) published in the literature. We also provide the computational time required by our method ($t(s)$) for completeness.

Table 6: Results for a set of VRPB benchmark instances.

| Instance | | | | | | BKS | BR-LNS | t(s) | Gap(%) |
|---|---|---|---|---|---|---|---|---|---|
| Name | N | n | m | Q | K | | | | |
| *Eil22_50* | 21 | 11 | 10 | 6000 | 3 | 371* | 371 | 0.1 | 0.00 |
| *Eil22_66* | 21 | 14 | 7 | 6000 | 3 | 366* | 366 | 0.2 | 0.00 |
| *Eil22_80* | 21 | 17 | 4 | 6000 | 3 | 375* | 375 | 0.2 | 0.00 |
| *Eil23_50* | 22 | 11 | 11 | 4500 | 2 | 682* | 682 | 0.5 | 0.00 |
| *Eil23_66* | 22 | 15 | 7 | 4500 | 2 | 649* | 649 | 0.3 | 0.00 |
| *Eil23_80* | 22 | 18 | 4 | 4500 | 2 | 623* | 623 | 0.5 | 0.00 |
| *Eil30_50* | 29 | 15 | 14 | 4500 | 2 | 501* | 501 | 0.7 | 0.00 |
| *Eil30_66* | 29 | 20 | 9 | 4500 | 3 | 537* | 537 | 0.2 | 0.00 |
| *Eil30_80* | 29 | 24 | 5 | 4500 | 3 | 514* | 514 | 0.4 | 0.00 |
| *Eil33_50* | 32 | 16 | 16 | 8000 | 3 | 738* | 738 | 1.1 | 0.00 |
| *Eil33_66* | 32 | 22 | 10 | 8000 | 3 | 750* | 750 | 0.8 | 0.00 |
| *Eil33_80* | 32 | 26 | 6 | 8000 | 3 | 736* | 736 | 0.1 | 0.00 |
| *Eil51_50* | 50 | 25 | 25 | 8000 | 3 | 559* | 559 | 1.9 | 0.00 |
| *Eil51_66* | 50 | 34 | 16 | 8000 | 4 | 548* | 548 | 1.4 | 0.00 |
| *Eil51_80* | 50 | 40 | 10 | 8000 | 4 | 565* | 565 | 1.5 | 0.00 |
| *EilA76_50* | 75 | 37 | 38 | 140 | 6 | 739* | 739 | 1.1 | 0.00 |
| *EilA76_66* | 75 | 50 | 25 | 140 | 7 | 768* | 768 | 2.9 | 0.00 |
| *EilA76_80* | 75 | 60 | 15 | 140 | 8 | 781 | 783 | 24.6 | 0.26 |
| *EilB76_50* | 75 | 37 | 38 | 100 | 8 | 801* | 801 | 3.7 | 0.00 |
| *EilB76_66* | 75 | 50 | 25 | 100 | 10 | 873* | 873 | 4.6 | 0.00 |
| *EilB76_80* | 75 | 60 | 15 | 100 | 12 | 919* | 919 | 10.6 | 0.00 |
| *EilC76_50* | 75 | 37 | 38 | 180 | 5 | 713* | 713 | 8.8 | 0.00 |
| *EilC76_66* | 75 | 50 | 25 | 180 | 6 | 734* | 734 | 2.6 | 0.00 |
| *EilC76_80* | 75 | 60 | 15 | 180 | 7 | 733 | 733 | 31.5 | 0.00 |
| *EilD76_50* | 75 | 37 | 38 | 220 | 4 | 690* | 690 | 20.4 | 0.00 |
| *EilD76_66* | 75 | 50 | 25 | 220 | 5 | 715 | 715 | 50.9 | 0.00 |
| *EilD76_80* | 75 | 60 | 15 | 220 | 6 | 694 | 694 | 15.7 | 0.00 |
| *EilA101_50* | 100 | 50 | 50 | 200 | 4 | 831 | 831 | 71.6 | 0.00 |
| *EilA101_66* | 100 | 67 | 33 | 200 | 6 | 846* | 846 | 13.2 | 0.00 |
| *EilA101_80* | 100 | 80 | 20 | 200 | 6 | 856 | 856 | 59.4 | 0.00 |
| *EilB101_50* | 100 | 50 | 50 | 112 | 7 | 923 | 925 | 39.5 | 0.22 |
| *EilB101_66* | 100 | 67 | 33 | 112 | 9 | 983 | 987 | 69.1 | 0.41 |
| *EilB101_80* | 100 | 80 | 20 | 112 | 11 | 1008 | 1009 | 17.9 | 0.10 |
| *Avg.* | | | | | | 700.6 | 700.9 | 13.9 | 0.03 |

* Optimal solution.

As can be observed, our results show an average gap of 0.03% with respect to the best-known solutions. In other words, despite our approach is designed to solve the 2L-VRPB, which includes both routing and two-dimensional packing, it also performs quite well in the routing-alone scenario.

Additionally, in order to show the efficiency of the packing procedure, we provide in Table 7 some examples of packing occupancy –measured as a percentage of the total loading area– for several instances and classes. In particular, column $O_{max}(\%)$ shows the maximum occupancy obtained for any route in the solution. In this case, considering the average occupancy might not be a representative value for the packing quality, since the routing process might require the vehicles to have unbalanced load distributions. As it can be observed in the provided examples, the packing procedure is able to reach very high occupancy levels, above 95% of the total loading surface.

Table 7: Examples of maximum occupancy levels ($O_{max}$) for several instances and classes of the 2L-VRPB.

| Linehaul / Backhaul | 2\|SO\|L | | | | | 2\|SO\|L | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Class | # | BR-LNS | K | $O_{max}$ (%) | Class | # | BR-LNS | K | $O_{max}$ (%) |
| 1/1 | 2 | 30 | 1415.14 | 17 | 96.6 | 3 | 10 | 521.00 | 3 | 94.5 |
| | 3 | 10 | 536.29 | 3 | 94.5 | 4 | 20 | 385.51 | 8 | 94.9 |
| | 5 | 36 | 946.13 | 22 | 95.5 | 5 | 28 | 1595.35 | 11 | 97.0 |
| 2/1 | 3 | 29 | 1722.74 | 17 | 94.5 | 3 | 14 | 857.36 | 4 | 93.6 |
| | 3 | 30 | 1554.94 | 22 | 95.3 | 3 | 24 | 912.60 | 11 | 95.4 |
| 4/1 | 3 | 14 | 879.84 | 4 | 96.3 | 2 | 35 | 1160.50 | 35 | 98.0 |
| | 4 | 35 | 1318.93 | 41 | 96.6 | 5 | 30 | 1406.47 | 19 | 96.6 |

## 6. Conclusions

This paper presents a hybrid approach for solving the two-dimensional VRP with clustered backhauls (2L-VRPB). This problem can be found in practical transportation applications, such as the one that motivated this work. To the best of our knowledge, this is the first time this realistic extension of the two-dimensional VRP (2L-VRP) is tackled in the literature. In addition, we permit items rotation when loading vehicles, a realistic assumption that has been rarely considered in previous works.

We propose a hybrid algorithm combining Large Neighbourhood Search with biased-randomised versions of well-known routing and packing heuristics. The search is controlled by the LNS framework, which partially destroys a solution by using splitting techniques and rebuilds it with a combination of skewed probability distributions to better guide the search. Packing heuristics are integrated in the routes construction, contrary to most two-stage methods from previous approaches. Our algorithm is also enhanced with memory-based techniques for both routing and packing processes, improving its execution performance without penalising the quality of the generated solutions.

As we have proved across different problem variants and loading configurations, our BR-LNS algorithm is able to provide high quality solutions in low computational times. In a number of cases, the proposed approach outperforms current state-of-the-art heuristics for the 2L-VRP, finding new best solutions for benchmark instances in generally much lower computational times. Hence, we conclude that the proposed approach constitutes not only the first method in reporting results for the 2L-VRPB, but also an efficient alternative to tackle the 2L-VRP, specially when realistic constraints such as sequential loading and items rotation are considered.

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## Appendix A. Detailed Computational Experiments

Table A.8: Results on the 36 instances of the 2L-VRP – Class 1 – both 2|SO|L and 2|SR|L

| # | VNS | t(s) | PRMP | t(s) | ACO | t(s) | BKS | BR-LNS | t(s) | Δ (%) |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 278.73 | 0.0 | 278.73 | 0.0 | 278.73 | 0.1 | 278.73 | 278.73 | 0.01 | 0.00 |
| 2 | 334.96 | 0.0 | 334.96 | 0.0 | 334.96 | 0.1 | 334.96 | 334.96 | 0.00 | 0.00 |
| 3 | 358.40 | 0.1 | 358.40 | 0.0 | 358.40 | 0.2 | 358.40 | 358.40 | 0.00 | 0.00 |
| 4 | 430.89 | 0.0 | 430.88 | 0.0 | 430.88 | 0.3 | 430.88 | 430.88 | 0.00 | 0.00 |
| 5 | 375.28 | 0.0 | 375.28 | 0.0 | 375.28 | 0.3 | 375.28 | 375.28 | 0.00 | 0.00 |
| 6 | 495.85 | 0.1 | 495.85 | 0.0 | 495.85 | 0.3 | 495.85 | 495.85 | 0.06 | 0.00 |
| 7 | 568.56 | 0.0 | 568.56 | 0.0 | 568.56 | 0.2 | 568.56 | 568.56 | 0.00 | 0.00 |
| 8 | 568.56 | 0.0 | 568.56 | 0.0 | 568.56 | 0.2 | 568.56 | 568.56 | 0.00 | 0.00 |
| 9 | 607.65 | 0.1 | 607.65 | 0.0 | 607.65 | 0.6 | 607.65 | 607.65 | 0.00 | 0.00 |
| 10 | 535.80 | 0.1 | 535.80 | 0.1 | 535.80 | 2.3 | **535.74** | 535.80 | 5.18 | 0.01 |
| 11 | 505.01 | 0.0 | 505.01 | 0.0 | 505.01 | 0.8 | 505.01 | 505.01 | 0.18 | 0.00 |
| 12 | 610.00 | 0.9 | 610.00 | 0.2 | 610.00 | 1.6 | 610.00 | 610.00 | 0.46 | 0.00 |
| 13 | 2006.34 | 0.1 | 2006.34 | 0.3 | 2006.34 | 1.3 | 2006.34 | 2006.34 | 0.08 | 0.00 |
| 14 | 837.67 | 0.1 | 837.67 | 0.1 | 837.67 | 4.1 | 837.67 | 837.67 | 0.20 | 0.00 |
| 15 | 837.67 | 0.1 | 837.67 | 0.4 | 837.67 | 2.8 | 837.67 | 837.67 | 0.18 | 0.00 |
| 16 | 698.61 | 1.1 | 698.61 | 0.3 | 698.61 | 2.0 | 698.61 | 698.61 | 0.16 | 0.00 |
| 17 | 861.79 | 4.0 | 861.79 | 1.6 | 861.79 | 3.3 | 861.79 | 861.79 | 185.65 | 0.00 |
| 18 | 723.54 | 1.4 | 723.54 | 3.6 | 723.54 | 9.5 | 723.54 | 723.54 | 1.10 | 0.00 |
| 19 | 524.61 | 2.0 | 524.61 | 2.1 | 524.61 | 7.9 | 524.61 | 524.61 | 7.29 | 0.00 |
| 20 | 241.97 | 3.5 | 241.97 | 7.2 | 241.97 | 55.7 | 241.97 | 241.97 | 2.85 | 0.00 |
| 21 | 687.60 | 74.9 | 687.60 | 3.8 | 690.20 | 26.7 | 687.60 | 687.60 | 164.12 | 0.00 |
| 22 | 740.66 | 21.2 | 740.66 | 2.8 | 742.91 | 56.9 | 740.66 | 740.66 | 12.63 | 0.00 |
| 23 | 835.26 | 159.7 | 835.26 | 48.7 | 845.34 | 55.9 | 835.26 | 835.26 | 30.74 | 0.00 |
| 24 | 1024.69 | 175.9 | 1024.69 | 38.1 | 1030.25 | 49.8 | 1024.69 | 1024.69 | 490.50 | 0.00 |
| 25 | 826.14 | 332.2 | 826.14 | 8.6 | 830.82 | 167.5 | 826.14 | 826.14 | 44.48 | 0.00 |
| 26 | 819.56 | 1.7 | 819.56 | 11.2 | 819.56 | 173.3 | 819.56 | 819.56 | 0.77 | 0.00 |
| 27 | 1082.65 | 445.5 | 1082.65 | 172.3 | 1100.22 | 191.0 | 1082.65 | 1082.65 | 9.50 | 0.00 |
| 28 | 1042.12 | 1021.5 | 1042.12 | 71.2 | 1062.23 | 252.2 | **1040.70** | 1042.12 | 136.28 | 0.14 |
| 29 | 1162.96 | 172.9 | 1162.96 | 121.9 | 1168.13 | 765.0 | 1162.96 | 1162.96 | 147.85 | 0.00 |
| 30 | 1028.42 | 1570.0 | 1028.42 | 267.5 | 1041.05 | 313.9 | 1028.42 | 1028.42 | 371.68 | 0.00 |
| 31 | 1302.48 | 1813.8 | 1299.56 | 353.8 | 1341.89 | 517.8 | 1299.56 | **1299.21** | 312.86 | -0.03 |
| 32 | 1300.22 | 1976.1 | 1296.91 | 312.0 | 1334.26 | 519.7 | 1296.91 | **1296.18** | 372.05 | -0.06 |
| 33 | 1298.02 | 2204.1 | 1299.55 | 434.1 | 1331.69 | 479.2 | 1298.02 | **1297.50** | 161.80 | -0.04 |
| 34 | **708.39** | 2125.2 | 709.82 | 328.2 | 712.32 | 621.4 | 708.39 | 709.08 | 554.20 | 0.10 |
| 35 | **865.39** | 2050.4 | 866.06 | 396.3 | 868.12 | 1468.2 | 865.39 | **864.63** | 382.43 | -0.09 |
| 36 | 586.49 | 2420.2 | **585.46** | 228.9 | 616.69 | 1589.8 | 585.46 | 590.16 | 560.74 | 0.80 |
| *Avg.* | 769.80 | 460.5 | 769.70 | 78.2 | 776.04 | 203.9 | 769.56 | 769.69 | 109.89 | 0.02 |

Table A.9: Results on the 36 instances of the 2L-VRP – Class 2 – 2|SO|L

| # | VNS | t(s) | PRMP | t(s) | ACO | t(s) | BKS | BR-LNS | t(s) | Δ (%) |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 290.84 | 3.3 | 290.84 | 0.5 | 290.84 | 8.9 | 290.84 | 290.84 | 0.05 | 0.00 |
| 2 | 347.73 | 0.4 | 347.73 | 0.5 | 347.73 | 0.1 | 347.73 | 347.73 | 0.00 | 0.00 |
| 3 | 403.93 | 0.9 | 403.93 | 0.6 | 403.93 | 1.8 | 403.93 | 403.93 | 0.01 | 0.00 |
| 4 | 440.94 | 0.4 | 440.94 | 0.6 | 440.94 | 1.3 | 440.94 | 440.94 | 0.01 | 0.00 |
| 5 | 388.72 | 2.4 | 388.72 | 2.3 | 388.72 | 10.8 | 388.72 | 388.72 | 0.03 | 0.00 |
| 6 | 499.08 | 0.5 | 499.08 | 4.3 | 499.08 | 2.8 | 499.08 | 499.08 | 0.03 | 0.00 |
| 7 | 734.65 | 1.8 | 734.65 | 4.8 | 734.65 | 6.1 | 734.65 | 734.65 | 0.98 | 0.00 |
| 8 | 725.91 | 16.0 | 725.91 | 7.1 | 725.91 | 5.9 | 725.91 | 725.91 | 0.10 | 0.00 |
| 9 | 611.49 | 1.2 | 611.49 | 6.2 | 611.49 | 1.9 | 611.49 | 611.49 | 0.01 | 0.00 |
| 10 | 700.20 | 12.0 | 700.20 | 52.8 | 700.20 | 40.3 | 700.20 | 700.20 | 3.74 | 0.00 |
| 11 | 721.54 | 10.0 | 721.54 | 86.8 | 721.54 | 34.7 | 721.54 | 723.34 | 11.30 | 0.25 |
| 12 | 619.63 | 5.7 | 619.63 | 5.1 | 619.63 | 5.6 | 619.63 | 619.63 | 0.24 | 0.00 |
| 13 | 2669.39 | 50.0 | 2669.39 | 103.0 | 2669.39 | 29.9 | 2669.39 | 2669.39 | 0.58 | 0.00 |
| 14 | 1111.94 | 488.0 | 1101.61 | 558.3 | 1135.93 | 106.0 | 1101.61 | **1090.55** | 360.01 | -1.00 |
| 15 | 1041.75 | 466.0 | 1099.91 | 64.5 | 1109.10 | 129.5 | 1041.75 | 1082.22 | 183.56 | 3.88 |
| 16 | 698.61 | 2.3 | 698.61 | 9.4 | 698.61 | 4.8 | 698.61 | 698.61 | 0.03 | 0.00 |
| 17 | 870.86 | 3.4 | 870.86 | 38.6 | 870.86 | 5.0 | 870.86 | 876.05 | 4.25 | 0.60 |
| 18 | 1053.09 | 394.0 | 1059.44 | 88.7 | 1059.44 | 191.9 | 1053.09 | 1059.74 | 183.91 | 0.63 |
| 19 | 792.42 | 319.0 | 794.15 | 175.3 | 794.75 | 68.2 | 792.42 | 792.58 | 118.31 | 0.02 |
| 20 | 547.82 | 1102.0 | 549.57 | 1647.4 | 553.12 | 1996.0 | 547.82 | 551.70 | 400.75 | 0.71 |
| 21 | 1060.72 | 941.0 | 1070.66 | 398.7 | 1076.95 | 2967.3 | 1060.72 | 1063.31 | 350.23 | 0.24 |
| 22 | 1081.44 | 1066.0 | 1086.25 | 510.1 | 1086.67 | 1193.4 | 1081.44 | 1081.45 | 486.38 | 0.00 |
| 23 | 1093.27 | 858.0 | 1113.50 | 594.5 | 1116.36 | 3632.1 | 1093.27 | 1112.93 | 363.47 | 1.80 |
| 24 | 1222.43 | 889.0 | 1222.43 | 894.4 | 1237.65 | 465.5 | 1222.43 | 1226.48 | 28.85 | 0.33 |
| 25 | 1458.83 | 1183.0 | 1476.34 | 1630.8 | 1478.83 | 2621.0 | 1458.83 | **1456.13** | 96.32 | -0.19 |
| 26 | 1327.47 | 1380.0 | 1330.94 | 1308.8 | 1332.40 | 2727.2 | 1327.47 | 1333.02 | 272.49 | 0.42 |
| 27 | 1367.85 | 1345.0 | 1367.87 | 2495.3 | 1376.49 | 3250.8 | 1367.85 | 1375.90 | 210.89 | 0.59 |
| 28 | 2699.21 | 2690.0 | 2717.14 | 9839.9 | 2726.26 | 10227.0 | 2699.21 | **2683.66** | 451.88 | -0.58 |
| 29 | 2289.84 | 2220.0 | 2309.35 | 5045.0 | 2328.5 | 10721.9 | 2289.84 | 2317.04 | 309.16 | 1.19 |
| 30 | 1875.38 | 2366.0 | 1915.54 | 4024.2 | 1899.53 | 9766.4 | 1875.38 | 1888.71 | 549.61 | 0.71 |
| 31 | 2369.07 | 2390.0 | 2389.26 | 7505.3 | 2382.29 | 10027.3 | 2369.07 | **2353.87** | 585.57 | -0.64 |
| 32 | 2384.29 | 2514.0 | 2413.19 | 9575.2 | 2419.54 | 10684.1 | 2384.29 | **2370.94** | 427.08 | -0.56 |
| 33 | 2376.58 | 2813.0 | 2415.80 | 4772.8 | 2431.53 | 10730.3 | 2376.58 | **2373.81** | 573.70 | -0.12 |
| 34 | 1226.98 | 2619.0 | 1253.52 | 10237.5 | 1265.01 | 10632.5 | 1226.98 | 1230.43 | 569.18 | 0.28 |
| 35 | 1447.30 | 3111.0 | 1491.44 | 7247.7 | 1504.61 | 10582.5 | 1447.30 | **1441.68** | 469.31 | -0.39 |
| 36 | 1784.57 | 3027.0 | 1810.07 | 9874.5 | 1858.33 | 10800.6 | 1784.57 | **1768.34** | 558.11 | -0.91 |
| *Avg.* | 1175.99 | 952.5 | 1186.43 | 2189.2 | 1191.58 | 3157.8 | 1175.71 | 1177.36 | 210.28 | 0.20 |

Table A.10: Results on the 36 instances of the 2L-VRP – Class 3 – 2|SO|L

| # | VNS | t(s) | PRMP | t(s) | ACO | t(s) | BKS | BR-LNS | t(s) | Δ (%) |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 284.52 | 11.1 | 284.52 | 0.5 | 304.41 | 17.0 | 284.52 | 301.88 | 0.32 | 6.10 |
| 2 | 352.16 | 1.6 | 352.16 | 0.5 | 356.24 | 0.7 | 352.16 | 352.16 | 0.85 | 0.00 |
| 3 | 394.72 | 1.5 | 394.72 | 0.7 | 394.72 | 3.8 | 394.72 | 394.72 | 0.22 | 0.00 |
| 4 | 440.68 | 0.5 | 440.68 | 0.7 | 445.25 | 3.4 | 440.68 | 440.68 | 3.59 | 0.00 |
| 5 | 381.69 | 9.0 | 381.69 | 2.7 | 381.69 | 20.3 | 381.69 | 381.69 | 0.02 | 0.00 |
| 6 | 504.68 | 2.5 | 504.68 | 5.2 | 504.68 | 4.7 | 504.68 | 504.68 | 0.03 | 0.00 |
| 7 | 709.72 | 5.2 | 709.72 | 5.8 | 709.72 | 11.1 | 709.72 | 709.72 | 0.70 | 0.00 |
| 8 | 741.12 | 1.9 | 741.12 | 7.9 | 741.12 | 12.3 | 741.12 | 741.12 | 1.12 | 0.00 |
| 9 | 613.90 | 1.5 | 619.48 | 5.0 | 613.90 | 9.4 | 613.90 | 619.48 | 0.06 | 0.91 |
| 10 | 637.38 | 123.0 | 646.87 | 66.6 | 628.93 | 56.8 | 628.93 | 637.46 | 97.60 | 1.36 |
| 11 | 717.37 | 136.0 | 720.45 | 68.4 | 718.09 | 38.6 | 717.37 | 720.63 | 81.51 | 0.45 |
| 12 | 610.00 | 1.6 | 610.00 | 4.6 | 610.00 | 2.3 | 610.00 | 610.00 | 0.29 | 0.00 |
| 13 | 2486.44 | 22.0 | 2504.78 | 98.6 | 2497.42 | 66.8 | 2486.44 | 2497.42 | 35.58 | 0.44 |
| 14 | 1085.42 | 533.0 | 1093.08 | 624.1 | 1093.63 | 160.6 | 1085.42 | **1069.43** | 81.19 | -1.47 |
| 15 | 1181.68 | 69.0 | 1181.68 | 65.6 | 1192.19 | 179.7 | 1181.68 | 1181.68 | 68.23 | 0.00 |
| 16 | 698.61 | 1.6 | 698.61 | 12.3 | 698.61 | 8.8 | 698.61 | 698.61 | 0.02 | 0.00 |
| 17 | 861.79 | 2.3 | 861.79 | 46.9 | 861.79 | 2.8 | 861.79 | 863.27 | 0.92 | 0.17 |
| 18 | 1103.45 | 251.0 | 1106.33 | 82.2 | 1120.55 | 189.6 | 1103.45 | 1115.67 | 11.39 | 1.11 |
| 19 | 801.13 | 243.0 | 802.76 | 181.3 | 801.13 | 145.5 | 801.13 | 802.76 | 4.71 | 0.20 |
| 20 | 541.58 | 961.0 | 546.63 | 2021.0 | 550.28 | 1308.5 | 541.58 | 546.11 | 295.78 | 0.84 |
| 21 | 1150.85 | 1036.0 | 1168.94 | 415.3 | 1162.07 | 438.8 | 1150.85 | 1157.22 | 144.69 | 0.55 |
| 22 | 1094.66 | 1264.0 | 1117.20 | 493.9 | 1123.10 | 536.7 | 1094.66 | 1099.97 | 249.67 | 0.49 |
| 23 | 1117.54 | 1130.0 | 1121.51 | 501.4 | 1141.01 | 531.0 | 1117.54 | 1120.41 | 187.32 | 0.26 |
| 24 | 1118.44 | 897.0 | 1128.30 | 947.7 | 1126.33 | 234.7 | 1118.44 | 1120.79 | 367.30 | 0.21 |
| 25 | 1436.57 | 1364.0 | 1452.95 | 2026.2 | 1459.58 | 4739.4 | 1436.57 | 1440.10 | 442.76 | 0.25 |
| 26 | 1396.52 | 1005.0 | 1411.53 | 1004.2 | 1409.10 | 1649.1 | 1396.52 | 1405.54 | 73.17 | 0.65 |
| 27 | 1423.74 | 1233.0 | 1449.27 | 4183.0 | 1450.35 | 1024.6 | 1423.74 | 1425.09 | 459.35 | 0.09 |
| 28 | 2787.24 | 2112.0 | 2800.34 | 9709.9 | 2796.83 | 10451.9 | 2787.24 | 2808.47 | 500.66 | 0.76 |
| 29 | 2172.69 | 3057.0 | 2201.64 | 4535.6 | 2231.33 | 8895.7 | 2172.69 | 2210.53 | 432.65 | 1.74 |
| 30 | 1915.42 | 2581.0 | 1947.51 | 4302.7 | 1980.40 | 10550.9 | 1915.42 | 1923.31 | 529.48 | 0.41 |
| 31 | 2360.63 | 2490.0 | 2413.28 | 5587.2 | 2425.91 | 10678.7 | 2360.63 | 2376.74 | 550.10 | 0.68 |
| 32 | 2325.74 | 2873.0 | 2386.89 | 10835.0 | 2400.24 | 10753.5 | 2325.74 | 2329.44 | 575.99 | 0.16 |
| 33 | 2469.85 | 2672.0 | 2535.42 | 5618.5 | 2522.99 | 10747.8 | 2469.85 | **2465.77** | 358.26 | -0.17 |
| 34 | 1253.88 | 2701.0 | 1278.94 | 12919.9 | 1302.24 | 10690.3 | 1253.88 | 1263.81 | 593.69 | 0.79 |
| 35 | 1529.77 | 2701.0 | 1566.63 | 9090.7 | 1576.22 | 10795.6 | 1529.77 | **1529.10** | 424.86 | -0.04 |
| 36 | 1869.38 | 3451.0 | 1889.21 | 7456.8 | 1952.47 | 10800.5 | 1869.38 | **1862.13** | 507.28 | -0.39 |
| *Avg.* | 1182.53 | 970.7 | 1196.43 | 2303.6 | 1202.35 | 2937.8 | 1182.29 | 1186.88 | 196.70 | 0.46 |

Table A.11: Results on the 36 instances of the 2L-VRP – Class 4 – 2|SO|L

| # | VNS | t(s) | PRMP | t(s) | ACO | t(s) | BKS | BR-LNS | t(s) | Δ (%) |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 294.25 | 0.5 | 294.25 | 1.8 | 296.75 | 0.9 | 294.25 | 294.25 | 0.09 | 0.00 |
| 2 | 342.00 | 0.2 | 342.00 | 1.4 | 342.00 | 0.3 | 342.00 | 342.00 | 0.01 | 0.00 |
| 3 | 368.56 | 2.0 | 368.56 | 1.5 | 368.56 | 4.6 | 368.56 | 368.56 | 0.08 | 0.00 |
| 4 | 447.37 | 2.6 | 447.37 | 2.8 | 447.37 | 4.5 | 447.37 | 447.37 | 0.02 | 0.00 |
| 5 | 383.88 | 4.4 | 383.87 | 2.5 | 383.88 | 14.3 | 383.87 | 383.87 | 0.06 | 0.00 |
| 6 | 498.32 | 3.0 | 498.32 | 5.2 | 498.32 | 6.9 | 498.32 | 498.32 | 0.19 | 0.00 |
| 7 | 703.49 | 14.0 | 703.49 | 4.5 | 703.49 | 13.9 | 703.49 | 703.49 | 1.00 | 0.00 |
| 8 | 697.92 | 31.0 | 697.92 | 6.1 | 733.09 | 8.6 | 697.92 | 697.92 | 14.94 | 0.00 |
| 9 | 625.10 | 11.6 | 625.10 | 7.0 | 625.13 | 4.4 | 625.10 | 625.13 | 0.04 | 0.00 |
| 10 | 715.82 | 189.0 | 715.82 | 50.8 | 760.61 | 56.6 | 715.82 | 717.83 | 51.23 | 0.28 |
| 11 | 815.68 | 41.0 | 815.68 | 59.1 | 816.10 | 104.1 | 815.68 | **811.56** | 67.33 | -0.51 |
| 12 | 618.23 | 17.0 | 618.23 | 8.6 | 623.20 | 16.0 | 618.23 | 618.23 | 11.65 | 0.00 |
| 13 | 2609.36 | 41.0 | 2610.57 | 106.6 | 2689.59 | 71.9 | 2609.36 | 2622.45 | 86.87 | 0.50 |
| 14 | 983.20 | 183.0 | 989.60 | 582.0 | 993.47 | 228.5 | 983.20 | 988.03 | 476.66 | 0.49 |
| 15 | 1246.49 | 449.0 | 1247.69 | 78.2 | 1276.85 | 148.3 | 1246.49 | 1246.69 | 289.46 | 0.02 |
| 16 | 708.20 | 8.0 | 708.20 | 14.1 | 709.27 | 12.3 | 708.20 | 712.30 | 0.12 | 0.58 |
| 17 | 861.79 | 7.6 | 861.79 | 37.3 | 861.79 | 4.6 | 861.79 | 861.79 | 33.11 | 0.00 |
| 18 | 1134.11 | 576.0 | 1137.34 | 91.8 | 1153.37 | 323.4 | 1134.11 | 1144.33 | 270.77 | 0.90 |
| 19 | 801.21 | 398.0 | 807.46 | 202.0 | 824.15 | 139.5 | 801.21 | 801.66 | 123.54 | 0.06 |
| 20 | 552.91 | 707.0 | 555.59 | 1591.9 | 567.72 | 642.4 | 552.91 | **551.61** | 157.95 | -0.23 |
| 21 | 1006.21 | 973.0 | 1014.17 | 372.5 | 1032.04 | 611.3 | 1006.21 | 1010.75 | 133.67 | 0.45 |
| 22 | 1089.27 | 1059.0 | 1101.93 | 518.7 | 1114.22 | 704.5 | 1089.27 | 1097.58 | 425.92 | 0.76 |
| 23 | 1093.01 | 678.0 | 1105.73 | 428.2 | 1110.62 | 1017.4 | 1093.01 | 1098.23 | 385.67 | 0.48 |
| 24 | 1141.97 | 991.0 | 1150.24 | 1087.9 | 1160.59 | 507.7 | 1141.97 | 1146.71 | 180.81 | 0.41 |
| 25 | 1435.18 | 1235.0 | 1467.34 | 2178.3 | 1480.83 | 1607.7 | 1435.18 | 1442.97 | 345.57 | 0.54 |
| 26 | 1447.03 | 1174.0 | 1468.43 | 1340.9 | 1582.50 | 5072.0 | 1447.03 | 1449.64 | 509.72 | 0.18 |
| 27 | 1357.75 | 1116.0 | 1371.88 | 4606.0 | 1404.41 | 1179.3 | 1357.75 | 1362.70 | 332.18 | 0.36 |
| 28 | 2700.66 | 1967.0 | 2731.04 | 8251.6 | 2765.90 | 10469.2 | 2700.66 | 2710.67 | 446.52 | 0.37 |
| 29 | 2312.37 | 2879.0 | 2340.96 | 4991.1 | 2390.83 | 10045.0 | 2312.37 | 2316.97 | 386.43 | 0.20 |
| 30 | 1910.54 | 2659.0 | 1946.53 | 4190.1 | 2020.59 | 10601.2 | 1910.54 | 1921.09 | 504.31 | 0.55 |
| 31 | 2469.40 | 2860.0 | 2523.98 | 5636.8 | 2611.01 | 10800.5 | 2469.40 | 2491.17 | 577.72 | 0.88 |
| 32 | 2357.57 | 2758.0 | 2410.15 | 8457.3 | 2518.80 | 10800.6 | 2357.57 | **2353.05** | 571.91 | -0.19 |
| 33 | 2470.76 | 2570.0 | 2532.13 | 5175.3 | 2627.36 | 10800.9 | 2470.76 | **2464.76** | 534.24 | -0.24 |
| 34 | 1242.26 | 3130.0 | 1279.67 | 14958.1 | 1324.63 | 10801.3 | 1242.26 | 1248.65 | 592.36 | 0.51 |
| 35 | 1558.69 | 3091.0 | 1599.02 | 10615.4 | 1890.84 | 10801.2 | 1558.69 | 1567.09 | 594.54 | 0.54 |
| 36 | 1740.64 | 3444.0 | 1768.88 | 9344.3 | 1843.85 | 10792.6 | 1740.64 | 1742.93 | 592.00 | 0.13 |
| *Avg.* | 1187.26 | 979.7 | 1201.14 | 2361.3 | 1237.60 | 3011.6 | 1187.26 | 1190.62 | 241.63 | 0.22 |

Table A.12: Results on the 36 instances of the 2L-VRP – Class 5 – 2|SO|L

| # | VNS | t(s) | PRMP | t(s) | ACO | t(s) | BKS | BR-LNS | t(s) | Δ (%) |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 278.73 | 3.2 | 278.73 | 2.7 | 285.93 | 0.7 | 278.73 | 278.73 | 0.25 | 0.00 |
| 2 | 334.96 | 0.0 | 334.96 | 1.7 | 334.96 | 0.2 | 334.96 | 334.96 | 0.02 | 0.00 |
| 3 | 358.40 | 1.4 | 358.40 | 2.2 | 358.40 | 1.3 | 358.40 | 358.40 | 0.00 | 0.00 |
| 4 | 430.89 | 0.3 | 430.88 | 2.4 | 430.88 | 2.3 | 430.88 | 430.88 | 0.00 | 0.00 |
| 5 | 375.28 | 0.8 | 375.28 | 2.7 | 375.28 | 2.9 | 375.28 | 375.28 | 0.02 | 0.00 |
| 6 | 495.85 | 0.9 | 495.85 | 7.6 | 495.85 | 1.8 | 495.85 | 495.85 | 0.00 | 0.00 |
| 7 | 658.64 | 28.0 | 661.22 | 6.0 | 661.22 | 14.5 | 658.64 | 658.64 | 17.34 | 0.00 |
| 8 | 621.85 | 35.9 | 633.23 | 6.9 | 646.46 | 20.8 | 621.85 | 646.46 | 0.04 | 3.96 |
| 9 | 607.65 | 0.6 | 607.65 | 6.4 | 607.65 | 3.6 | 607.65 | 607.65 | 0.01 | 0.00 |
| 10 | 690.96 | 139.0 | 691.04 | 49.9 | 699.05 | 83.0 | 690.96 | 694.71 | 84.70 | 0.54 |
| 11 | 636.77 | 31.0 | 645.65 | 87.0 | 658.71 | 97.6 | 636.77 | 642.20 | 114.41 | 0.85 |
| 12 | 610.23 | 5.8 | 610.23 | 10.2 | 610.23 | 6.8 | 610.23 | 610.23 | 0.12 | 0.00 |
| 13 | 2421.88 | 101.0 | 2434.99 | 170.2 | 2510.64 | 124.3 | 2421.88 | 2434.99 | 10.91 | 0.54 |
| 14 | 924.27 | 462.0 | 925.21 | 783.8 | 946.31 | 275.1 | 924.27 | 943.02 | 84.47 | 2.03 |
| 15 | 1230.40 | 210.0 | 1230.60 | 64.4 | 1273.60 | 209.1 | 1230.40 | 1230.37 | 150.70 | 0.00 |
| 16 | 698.61 | 1.7 | 698.61 | 20.8 | 698.61 | 5.2 | 698.61 | 698.61 | 0.06 | 0.00 |
| 17 | 861.79 | 4.3 | 861.79 | 40.6 | 861.79 | 4.7 | 861.79 | 861.79 | 0.24 | 0.00 |
| 18 | 926.53 | 365.0 | 928.88 | 117.8 | 939.20 | 711.3 | 926.53 | 928.87 | 68.40 | 0.25 |
| 19 | 652.58 | 230.0 | 659.84 | 194.7 | 676.32 | 232.0 | 652.58 | 658.01 | 264.26 | 0.83 |
| 20 | 478.73 | 917.0 | 487.68 | 1383.1 | 495.51 | 1281.3 | 478.73 | 490.80 | 473.02 | 2.52 |
| 21 | 893.18 | 1062.0 | 913.32 | 494.1 | 925.94 | 1092.4 | 893.18 | 910.31 | 321.83 | 1.92 |
| 22 | 948.60 | 1042.0 | 960.87 | 574.3 | 982.34 | 1141.2 | 948.60 | 959.18 | 301.21 | 1.12 |
| 23 | 950.25 | 1146.0 | 964.02 | 553.9 | 984.19 | 750.6 | 950.25 | 964.16 | 487.16 | 1.46 |
| 24 | 1048.69 | 588.0 | 1055.51 | 1327.2 | 1078.62 | 305.6 | 1048.69 | 1049.76 | 51.82 | 0.10 |
| 25 | 1183.63 | 1442.0 | 1207.36 | 3442.6 | 1233.93 | 2984.3 | 1183.63 | 1202.25 | 397.26 | 1.57 |
| 26 | 1252.65 | 1402.0 | 1272.21 | 2310.7 | 1304.02 | 2230.3 | 1252.65 | 1261.07 | 467.41 | 0.67 |
| 27 | 1270.34 | 1275.0 | 1300.94 | 5371.0 | 1326.58 | 2037.6 | 1270.34 | 1294.92 | 565.17 | 1.94 |
| 28 | 2399.25 | 2924.0 | 2427.73 | 6759.0 | 2485.45 | 10334.4 | 2399.25 | 2423.74 | 599.90 | 1.02 |
| 29 | 2191.69 | 2535.0 | 2203.11 | 7365.5 | 2273.96 | 10401.9 | 2191.69 | 2197.37 | 578.39 | 0.26 |
| 30 | 1575.64 | 2402.0 | 1602.51 | 6190.5 | 1643.68 | 10098.9 | 1575.64 | 1597.44 | 502.74 | 1.38 |
| 31 | 2072.19 | 3303.0 | 2106.52 | 4652.2 | 2149.12 | 10528.5 | 2072.19 | 2073.95 | 561.74 | 0.08 |
| 32 | 2031.92 | 2511.0 | 2080.61 | 8865.4 | 2121.11 | 10674.7 | 2031.92 | 2055.56 | 572.90 | 1.16 |
| 33 | 2054.29 | 2403.0 | 2093.93 | 7083.5 | 2159.01 | 10395.9 | 2054.29 | 2073.33 | 565.30 | 0.93 |
| 34 | 1062.18 | 2853.0 | 1090.00 | 14451.7 | 1114.58 | 10475.5 | 1062.18 | 1082.77 | 516.88 | 1.94 |
| 35 | 1281.90 | 3309.0 | 1320.18 | 9004.5 | 1357.62 | 10448.6 | 1281.90 | 1313.31 | 586.02 | 2.45 |
| 36 | 1549.51 | 3208.0 | 1580.52 | 13562.9 | 1658.81 | 10801.5 | 1549.51 | 1567.06 | 597.96 | 1.13 |
| *Avg.* | 1057.25 | 998.4 | 1070.28 | 2638.1 | 1093.49 | 2993.9 | 1057.25 | 1066.85 | 248.41 | 0.85 |

Table A.13: Results on the 36 instances of the 2L-VRP – 2|SR|L

| # | Class 2 | | | | Class 3 | | | | Class 4 | | | | Class 5 | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | ACO | t(s) | BR-LNS | t(s) | ACO | t(s) | BR-LNS | t(s) | ACO | t(s) | BR-LNS | t(s) | ACO | t(s) | BR-LNS | t(s) |
| 1 | 278.73 | 4.3 | 278.73 | 0.01 | 284.52 | 9.0 | **284.23** | 19.90 | 282.95 | 0.4 | 282.95 | 0.00 | 280.60 | 0.5 | **278.73** | 0.43 |
| 2 | 334.96 | 0.1 | 334.96 | 0.01 | 352.16 | 0.4 | 352.16 | 0.38 | 342.00 | 0.2 | **334.96** | 31.87 | 334.96 | 0.1 | 334.96 | 0.11 |
| 3 | 384.93 | 2.0 | 384.93 | 6.37 | 394.72 | 2.4 | **390.55** | 23.80 | 368.56 | 1.0 | **362.41** | 10.13 | 358.40 | 0.7 | 358.40 | 0.02 |
| 4 | 430.88 | 2.5 | 430.88 | 0.05 | 430.88 | 1.5 | 430.88 | 0.83 | 447.37 | 2.9 | 447.37 | 0.07 | 430.88 | 1.5 | 430.88 | 0.00 |
| 5 | 375.28 | 13.1 | 375.28 | 0.21 | 379.94 | 12.8 | 379.94 | 14.14 | 383.88 | 13.3 | 383.87 | 0.23 | 375.28 | 3.7 | 375.28 | 0.06 |
| 6 | 498.16 | 2.3 | **495.85** | 11.28 | 498.16 | 3.1 | 498.16 | 0.03 | 498.32 | 4.7 | 498.32 | 0.35 | 495.85 | 2.1 | 495.85 | 0.00 |
| 7 | 716.82 | 5.0 | 716.82 | 30.11 | 706.99 | 3.2 | **678.75** | 43.67 | 702.45 | 11.6 | **686.26** | 3.91 | 658.64 | 14.9 | 658.64 | 2.23 |
| 8 | 674.20 | 3.6 | **671.75** | 8.14 | 741.12 | 7.1 | 741.12 | 0.80 | 705.89 | 13.3 | **692.47** | 23.29 | 639.18 | 28.4 | 640.01 | 47.36 |
| 9 | 607.65 | 2.1 | 607.65 | 0.01 | 607.65 | 4.3 | 607.65 | 0.05 | 625.13 | 4.2 | **625.10** | 0.35 | 607.65 | 2.8 | 607.65 | 0.02 |
| 10 | 685.21 | 39.9 | **684.37** | 184.09 | 617.62 | 44.8 | **615.68** | 3.07 | 722.70 | 41.6 | **708.68** | 97.39 | 693.15 | 83.2 | 691.04 | 132.01 |
| 11 | 694.60 | 27.6 | 702.74 | 85.28 | 706.73 | 27.6 | 711.80 | 85.10 | 800.88 | 37.6 | **777.04** | 48.58 | 644.46 | 102.2 | 644.93 | 132.61 |
| 12 | 615.87 | 5.5 | **610.00** | 0.38 | 610.23 | 2.4 | **610.00** | 0.42 | 619.21 | 7.8 | **614.23** | 45.18 | 610.23 | 5.2 | 610.23 | 0.57 |
| 13 | 2526.07 | 42.8 | 2534.97 | 36.25 | 2469.98 | 41.9 | **2436.06** | 98.16 | 2623.65 | 70.4 | **2605.34** | 14.67 | 2434.99 | 114.2 | **2386.96** | 176.08 |
| 14 | 1041.61 | 107.0 | **1032.01** | 93.99 | 1012.46 | 143.0 | **1009.46** | 156.35 | 988.25 | 174.6 | **982.16** | 169.49 | 927.79 | 258.3 | **922.02** | 497.21 |
| 15 | 1009.87 | 94.9 | 1009.87 | 191.21 | 1170.82 | 88.2 | **1142.18** | 236.84 | 1245.94 | 132.7 | **1171.41** | 88.04 | 1234.87 | 219.4 | **1204.30** | 299.46 |
| 16 | 698.61 | 5.6 | 698.61 | 0.04 | 698.61 | 5.6 | 698.61 | 0.09 | 703.35 | 10.7 | 703.35 | 1.55 | 698.61 | 5.8 | 698.61 | 0.06 |
| 17 | 861.79 | 4.0 | 861.79 | 1.51 | 861.79 | 2.9 | 861.79 | 3.45 | 861.79 | 4.5 | 861.79 | 0.17 | 861.79 | 4.4 | 861.79 | 4.27 |
| 18 | 989.21 | 171.9 | **988.36** | 284.36 | 1031.94 | 154.2 | **1030.69** | 15.00 | 1128.25 | 289.8 | **1104.08** | 168.47 | 926.39 | 579.1 | **925.72** | 122.50 |
| 19 | 732.64 | 54.4 | **731.93** | 3.93 | 757.59 | 67.4 | 764.14 | 115.74 | 796.42 | 95.6 | **772.39** | 192.33 | 664.28 | 208.8 | **653.92** | 210.05 |
| 20 | 496.93 | 369.0 | 500.98 | 139.86 | 536.58 | 444.2 | **519.15** | 564.11 | 549.38 | 468.8 | **542.90** | 504.26 | 488.28 | 1058.5 | **478.37** | 563.48 |
| 21 | 998.48 | 363.8 | **988.25** | 122.29 | 1126.49 | 312.5 | **1102.63** | 386.65 | 1006.61 | 576.0 | **976.38** | 272.83 | 911.01 | 1002.4 | **892.18** | 416.27 |
| 22 | 1009.25 | 300.5 | **1000.79** | 482.33 | 1060.79 | 361.3 | **1030.00** | 327.47 | 1089.59 | 416.1 | **1062.94** | 66.92 | 956.71 | 1111.6 | **946.90** | 233.34 |
| 23 | 1019.73 | 386.5 | **997.58** | 386.89 | 1090.74 | 350.5 | **1069.58** | 469.79 | 1098.82 | 456.2 | **1077.98** | 438.92 | 959.79 | 662.8 | **949.49** | 389.23 |
| 24 | 1183.02 | 184.4 | **1174.29** | 283.20 | 1091.54 | 171.0 | **1076.30** | 193.22 | 1129.79 | 231.3 | **1101.83** | 404.89 | 1059.06 | 252.6 | **1047.80** | 240.72 |
| 25 | 1383.57 | 727.0 | **1373.52** | 267.91 | 1374.51 | 817.2 | **1363.21** | 155.40 | 1442.71 | 1035.2 | **1404.84** | 387.62 | 1204.57 | 3036.1 | **1183.77** | 495.82 |
| 26 | 1283.32 | 827.3 | **1275.83** | 344.82 | 1372.46 | 942.3 | **1341.19** | 385.36 | 1450.79 | 3165.3 | **1406.40** | 553.11 | 1260.75 | 2268.3 | **1254.96** | 457.81 |
| 27 | 1310.13 | 692.5 | **1295.03** | 384.60 | 1399.26 | 670.0 | **1369.42** | 482.52 | 1369.55 | 1053.3 | **1317.17** | 439.13 | 1297.83 | 2006.8 | **1259.00** | 434.07 |
| 28 | 2548.28 | 8804.8 | **2528.53** | 569.58 | 2710.90 | 9072.4 | **2583.70** | 500.81 | 2724.74 | 10403.5 | **2655.25** | 571.45 | 2416.88 | 10246.3 | **2358.10** | 579.54 |
| 29 | 2197.20 | 9900.3 | **2168.94** | 587.86 | 2138.30 | 10061.1 | **2084.66** | 343.38 | 2327.21 | 10058.5 | **2274.96** | 599.13 | 2206.55 | 9829.6 | **2165.02** | 471.11 |
| 30 | 1805.65 | 7913.3 | **1770.80** | 387.58 | 1874.66 | 9269.0 | **1817.72** | 305.17 | 1908.00 | 9902.4 | **1840.99** | 524.88 | 1593.81 | 10671.9 | **1558.68** | 547.27 |
| 31 | 2265.21 | 10493.5 | **2223.03** | 581.53 | 2326.58 | 10255.1 | **2264.96** | 593.78 | 2486.10 | 9858.1 | **2392.25** | 571.64 | 2097.62 | 10801.0 | **2040.29** | 589.48 |
| 32 | 2258.99 | 10489.3 | **2229.46** | 278.58 | 2305.18 | 9855.6 | **2235.42** | 581.38 | 2372.77 | 10711.3 | **2282.58** | 560.04 | 2062.45 | 10119.3 | **2004.86** | 584.30 |
| 33 | 2251.88 | 10415.5 | **2208.80** | 443.98 | 2407.87 | 9942.3 | **2343.34** | 581.32 | 2479.42 | 10550.2 | **2389.32** | 526.16 | 2081.18 | 9873.2 | **2018.52** | 563.98 |
| 34 | 1172.76 | 10082.4 | **1157.28** | 367.87 | 1238.94 | 10479.5 | **1193.34** | 551.68 | 1251.31 | 10767.9 | **1211.21** | 462.08 | 1088.21 | 10511.5 | **1055.12** | 579.36 |
| 35 | 1375.14 | 10460.2 | **1362.09** | 384.25 | 1483.13 | 10497.0 | **1444.20** | 595.51 | 1606.54 | 10800.9 | **1511.27** | 511.50 | 1314.60 | 10772.6 | **1285.15** | 471.38 |
| 36 | 1737.64 | 10688.4 | **1680.64** | 566.70 | 1842.36 | 10723.8 | **1769.88** | 536.11 | 1765.72 | 10658.8 | **1673.25** | 598.41 | 1594.14 | 10623.9 | **1527.63** | 589.34 |
| *Avg.* | 1123.73 | 2602.4 | 1113.54 | 208.81 | 1158.73 | 2634.6 | 1134.79 | 232.54 | 1191.83 | 2834.2 | 1159.33 | 246.92 | 1068.65 | 2957.9 | 1050.16 | 273.10 |

Table A.14: Results on the 36 instances of the 2L-VRPB – Class 1 – 2|SO|L and 2|SR|L

| # | Line/Back: 1/1 | | | Line/Back: 2/1 | | | Line/Back: 4/1 | | |
|---|---|---|---|---|---|---|---|---|---|
| | BR-LNS | K | t(s) | BR-LNS | K | t(s) | BR-LNS | K | t(s) |
| 1 | 301.99 | 2 | 7.34 | 274.25 | 3 | 0.00 | 259.97 | 3 | 0.00 |
| 2 | 308.76 | 3 | 0.01 | 323.52 | 5 | 0.00 | 299.64 | 5 | 0.00 |
| 3 | 335.54 | 3 | 0.00 | 352.70 | 3 | 0.01 | 349.12 | 4 | 0.00 |
| 4 | 375.12 | 5 | 0.00 | 396.11 | 6 | 0.00 | 415.83 | 6 | 0.00 |
| 5 | 372.12 | 3 | 0.01 | 365.55 | 3 | 0.00 | 376.68 | 4 | 0.00 |
| 6 | 432.30 | 4 | 0.01 | 405.99 | 4 | 0.00 | 432.83 | 5 | 0.00 |
| 7 | 689.32 | 3 | 6.30 | 678.88 | 3 | 1.06 | 598.68 | 3 | 0.00 |
| 8 | 689.32 | 3 | 2.52 | 692.49 | 3 | 4.00 | 598.68 | 3 | 0.00 |
| 9 | 494.03* | 5 | 1.50 | 526.48 | 6 | 0.00 | 571.75 | 7 | 0.18 |
| 10 | 502.77 | 3 | 2.42 | 550.62 | 3 | 1.45 | 512.06 | 3 | 0.02 |
| 11 | 502.77 | 3 | 2.18 | 550.62 | 3 | 1.04 | 512.06 | 3 | 0.13 |
| 12 | 471.46 | 6 | 0.00 | 497.63 | 7 | 0.09 | 523.41 | 7 | 0.01 |
| 13 | 2276.57 | 3 | 4.63 | 2184.46 | 3 | 41.64 | 1997.84 | 3 | 0.04 |
| 14 | 751.69 | 3 | 0.17 | 725.95 | 3 | 2.41 | 746.28 | 3 | 0.81 |
| 15 | 751.69 | 3 | 0.18 | 725.95 | 3 | 2.41 | 746.28 | 3 | 0.80 |
| 16 | 543.09 | 6 | 0.02 | 582.64 | 8 | 0.03 | 613.19 | 9 | 0.01 |
| 17 | 638.14 | 9 | 0.01 | 697.42 | 10 | 0.10 | 725.83 | 12 | 0.01 |
| 18 | 834.86 | 4 | 1.06 | 814.27 | 3 | 66.98 | 791.40 | 4 | 2.34 |
| 19 | 562.83 | 3 | 49.32 | 578.70 | 4 | 49.29 | 567.89 | 5 | 0.57 |
| 20 | 319.72 | 4 | 71.97 | 304.45 | 4 | 32.38 | 288.90 | 4 | 94.58 |
| 21 | 721.78 | 5 | 52.93 | 715.42 | 5 | 27.74 | 703.81 | 5 | 111.97 |
| 22 | 721.68 | 5 | 32.36 | 742.14 | 6 | 29.05 | 733.42 | 6 | 3.33 |
| 23 | 746.90 | 6 | 89.69 | 773.12 | 7 | 5.81 | 794.85 | 8 | 5.37 |
| 24 | 838.96 | 9 | 0.31 | 873.83 | 10 | 0.03 | 904.53 | 11 | 1.15 |
| 25 | 889.59 | 6 | 183.73 | 830.07 | 5 | 78.20 | 859.97 | 7 | 4.40 |
| 26 | 779.21 | 7 | 51.93 | 773.24 | 7 | 165.33 | 833.59 | 8 | 115.71 |
| 27 | 964.88 | 9 | 175.88 | 974.54 | 10 | 3.07 | 1004.20 | 11 | 130.34 |
| 28 | 1022.91 | 5 | 215.51 | 1039.50 | 6 | 162.41 | 1059.68 | 6 | 145.08 |
| 29 | 1217.36* | 6 | 241.36 | 1342.38 | 7 | 201.99 | 1210.77 | 7 | 220.03 |
| 30 | 1050.11 | 8 | 219.99 | 1059.28 | 9 | 98.42 | 1067.26 | 11 | 52.52 |
| 31 | 1216.24 | 11 | 233.28 | 1278.37 | 12 | 154.73 | 1260.15 | 14 | 127.00 |
| 32 | 1202.83 | 10 | 132.09 | 1291.09 | 12 | 193.06 | 1260.15 | 14 | 126.56 |
| 33 | 1213.71 | 11 | 236.67 | 1305.80 | 13 | 189.83 | 1295.28 | 14 | 118.13 |
| 34 | 702.84 | 18 | 271.66 | 633.81 | 16 | 207.33 | 654.71 | 17 | 197.28 |
| 35 | 747.01 | 16 | 269.40 | 793.11 | 19 | 173.90 | 839.02 | 22 | 109.90 |
| 36 | 488.96 | 8 | 95.28 | 550.55 | 10 | 95.85 | 584.62 | 12 | 283.14 |

* In these instances, linehaul and backhaul customer set were swapped, to avoid $K_L < K_B$.

Table A.15: Results on the 36 instances of the 2L-VRPB – Class 2

| # | 2\|SO\|L | | | | | | | | | 2\|SR\|L | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Line/Back: 1/1 | | | Line/Back: 2/1 | | | Line/Back: 4/1 | | | Line/Back: 1/1 | | | Line/Back: 2/1 | | | Line/Back: 4/1 | | |
| | BR-LNS | K | t(s) | BR-LNS | K | t(s) | BR-LNS | K | t(s) | BR-LNS | K | t(s) | BR-LNS | K | t(s) | BR-LNS | K | t(s) |
| 1 | 308.76 | 3 | 0.16 | 274.25 | 3 | 0.00 | 259.97 | 3 | 0.00 | 307.63 | 3 | 0.03 | 274.25 | 3 | 0.00 | 259.97 | 3 | 0.01 |
| 2 | 308.76 | 3 | 0.00 | 323.52 | 5 | 0.00 | 314.14 | 5 | 0.00 | 308.76 | 3 | 0.00 | 323.52 | 5 | 0.00 | 299.64 | 5 | 0.00 |
| 3 | 336.40 | 3 | 0.04 | 380.89 | 4 | 0.01 | 350.83 | 4 | 0.00 | 335.54 | 3 | 0.00 | 358.42 | 4 | 0.00 | 349.12 | 4 | 0.01 |
| 4 | 375.12 | 5 | 0.01 | 397.66 | 5 | 0.00 | 395.42 | 5 | 0.00 | 375.12 | 5 | 0.00 | 397.66 | 5 | 0.00 | 395.42 | 5 | 0.01 |
| 5 | 376.84 | 3 | 0.13 | 378.70 | 3 | 0.05 | 376.68 | 4 | 0.00 | 372.12 | 3 | 0.01 | 365.55 | 3 | 0.09 | 376.68 | 4 | 0.00 |
| 6 | 428.88 | 4 | 0.07 | 408.53 | 4 | 0.00 | 432.85 | 5 | 0.00 | 428.88 | 4 | 0.07 | 405.99 | 4 | 0.00 | 432.83 | 5 | 0.02 |
| 7 | 692.26 | 4 | 0.01 | 703.67 | 4 | 0.00 | 723.39 | 5 | 1.29 | 689.32 | 3 | 0.73 | 703.67 | 4 | 0.00 | 694.63 | 4 | 0.02 |
| 8 | 698.87 | 4 | 0.00 | 693.58 | 4 | 0.05 | 683.64 | 4 | 0.01 | 689.32 | 3 | 3.73 | 693.58 | 4 | 0.00 | 663.45 | 4 | 0.02 |
| 9 | 501.48 | 5 | 0.01 | 531.24 | 6 | 0.02 | 573.06 | 8 | 0.00 | 494.03 | 5 | 1.91 | 526.48 | 6 | 0.02 | 571.75 | 7 | 0.06 |
| 10 | 610.45 | 4 | 0.08 | 611.94 | 4 | 6.14 | 642.58 | 5 | 4.31 | 556.61 | 3 | 0.67 | 593.98 | 4 | 0.10 | 613.00 | 4 | 0.33 |
| 11 | 603.37 | 4 | 0.27 | 626.35 | 5 | 4.02 | 662.43 | 5 | 2.10 | 573.79 | 4 | 8.76 | 611.00 | 5 | 0.38 | 632.01 | 5 | 28.02 |
| 12 | 482.63 | 6 | 0.04 | 515.80 | 7 | 0.53 | 546.33 | 8 | 0.06 | 476.08 | 6 | 0.00 | 505.52 | 7 | 0.05 | 532.12 | 8 | 0.04 |
| 13 | 2399.98 | 4 | 1.28 | 2407.68 | 5 | 3.81 | 2489.25 | 5 | 8.45 | 2360.14 | 4 | 13.89 | 2345.91 | 4 | 8.06 | 2324.70 | 5 | 1.47 |
| 14 | 870.04 | 4 | 17.33 | 894.55 | 4 | 0.87 | 1017.55 | 5 | 3.15 | 783.87 | 3 | 48.44 | 877.66 | 4 | 0.05 | 902.70 | 4 | 45.11 |
| 15 | 850.73 | 3 | 1.00 | 834.07 | 4 | 15.72 | 963.49 | 5 | 4.84 | 786.21 | 3 | 20.65 | 761.96 | 3 | 56.38 | 887.45 | 4 | 29.40 |
| 16 | 549.86 | 7 | 0.69 | 578.20 | 8 | 0.03 | 614.67 | 9 | 0.02 | 542.60 | 7 | 0.13 | 578.20 | 8 | 0.14 | 609.85 | 9 | 0.83 |
| 17 | 635.94 | 8 | 0.11 | 681.87 | 9 | 0.46 | 734.15 | 12 | 0.07 | 635.94 | 8 | 0.06 | 680.30 | 9 | 0.22 | 727.90 | 12 | 37.01 |
| 18 | 937.03 | 5 | 54.06 | 956.22 | 6 | 11.81 | 1000.84 | 7 | 3.21 | 898.99 | 5 | 6.48 | 905.48 | 5 | 43.24 | 947.31 | 6 | 19.62 |
| 19 | 655.44 | 6 | 2.79 | 694.60 | 8 | 43.48 | 698.50 | 8 | 57.67 | 616.69 | 6 | 2.81 | 647.53 | 7 | 9.56 | 669.58 | 8 | 87.67 |
| 20 | 419.92 | 9 | 93.00 | 427.23 | 10 | 25.86 | 460.16 | 11 | 92.13 | 372.83 | 8 | 66.40 | 388.27 | 9 | 63.96 | 435.36 | 10 | 77.42 |
| 21 | 876.37 | 9 | 21.22 | 927.84 | 11 | 60.83 | 965.26 | 13 | 70.23 | 801.36 | 7 | 81.76 | 870.89 | 10 | 120.77 | 900.67 | 11 | 28.91 |
| 22 | 872.10 | 9 | 98.03 | 883.66 | 10 | 24.97 | 990.59 | 14 | 31.10 | 822.11 | 8 | 12.37 | 853.83 | 10 | 5.84 | 936.02 | 12 | 75.92 |
| 23 | 880.09 | 9 | 74.67 | 931.68 | 11 | 76.38 | 958.56 | 12 | 82.44 | 832.02 | 8 | 102.58 | 850.40 | 10 | 73.93 | 886.38 | 11 | 96.83 |
| 24 | 920.51 | 10 | 105.86 | 1007.87 | 12 | 19.51 | 1061.97 | 14 | 60.80 | 889.85 | 10 | 1.42 | 963.74 | 12 | 12.58 | 1022.57 | 14 | 35.61 |
| 25 | 1144.05 | 12 | 20.28 | 1219.02 | 15 | 46.83 | 1312.22 | 18 | 127.47 | 1060.56 | 11 | 105.06 | 1136.68 | 14 | 133.80 | 1227.70 | 17 | 70.87 |
| 26 | 1031.22 | 12 | 36.97 | 1095.65 | 13 | 92.23 | 1259.05 | 17 | 54.06 | 956.30 | 11 | 142.23 | 1051.25 | 12 | 119.30 | 1173.86 | 15 | 156.54 |
| 27 | 1073.48 | 12 | 157.05 | 1173.29 | 14 | 189.14 | 1245.97 | 16 | 18.04 | 1026.46 | 11 | 68.44 | 1108.04 | 13 | 101.54 | 1164.18 | 15 | 176.25 |
| 28 | 1780.33 | 13 | 120.89 | 1925.05 | 15 | 239.92 | 2303.88 | 18 | 242.63 | 1671.31 | 12 | 247.25 | 1822.48 | 14 | 217.61 | 2139.68 | 17 | 169.05 |
| 29 | 1727.00 | 14 | 203.58 | 1846.64 | 17 | 235.15 | 2009.95 | 20 | 165.31 | 1572.62 | 13 | 238.06 | 1765.09 | 16 | 146.91 | 1935.64 | 19 | 248.90 |
| 30 | 1415.14 | 17 | 101.35 | 1560.45 | 20 | 259.60 | 1687.05 | 24 | 271.49 | 1312.54 | 15 | 200.60 | 1453.13 | 19 | 261.81 | 1585.44 | 23 | 262.17 |
| 31 | 1686.66 | 21 | 115.50 | 1904.79 | 27 | 242.69 | 2048.22 | 31 | 130.41 | 1589.06 | 20 | 144.63 | 1785.43 | 25 | 271.58 | 1955.70 | 29 | 241.67 |
| 32 | 1700.82 | 22 | 262.50 | 1894.47 | 26 | 98.30 | 2038.96 | 31 | 273.48 | 1584.83 | 20 | 265.98 | 1778.63 | 24 | 233.42 | 1877.14 | 28 | 207.20 |
| 33 | 1716.05 | 22 | 272.15 | 1905.83 | 26 | 203.21 | 2081.76 | 31 | 251.38 | 1609.70 | 20 | 237.06 | 1764.91 | 24 | 243.89 | 1947.88 | 28 | 272.65 |
| 34 | 890.10 | 28 | 236.21 | 953.01 | 32 | 288.21 | 1066.72 | 38 | 185.72 | 825.33 | 24 | 246.89 | 898.38 | 30 | 262.28 | 1003.07 | 36 | 278.65 |
| 35 | 1006.72 | 26 | 197.76 | 1133.30 | 32 | 212.57 | 1253.38 | 38 | 266.50 | 897.77 | 23 | 277.47 | 1054.42 | 30 | 260.07 | 1160.50 | 35 | 295.64 |
| 36 | 1090.58 | 25 | 262.33 | 1267.03 | 32 | 283.60 | 1500.73 | 39 | 278.22 | 1016.32 | 24 | 249.13 | 1204.45 | 30 | 298.01 | 1401.12 | 36 | 298.13 |

Table A.16: Results on the 36 instances of the 2L-VRPB – Class 3

| # | 2\|SO\|L | | | | | | | | | 2\|SR\|L | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Line/Back: 1/1 | | | Line/Back: 2/1 | | | Line/Back: 4/1 | | | Line/Back: 1/1 | | | Line/Back: 2/1 | | | Line/Back: 4/1 | | |
| | BR-LNS | K | t(s) | BR-LNS | K | t(s) | BR-LNS | K | t(s) | BR-LNS | K | t(s) | BR-LNS | K | t(s) | BR-LNS | K | t(s) |
| 1 | 308.76 | 3 | 0.01 | 274.25 | 3 | 0.00 | 260.22 | 3 | 0.01 | 307.63 | 3 | 0.01 | 274.25 | 3 | 0.00 | 259.97 | 3 | 0.01 |
| 2 | 308.76 | 3 | 0.01 | 323.52 | 5 | 0.00 | 322.42 | 5 | 0.00 | 308.76 | 3 | 0.00 | 323.52 | 5 | 0.00 | 310.97 | 5 | 0.00 |
| 3 | 345.66 | 4 | 0.14 | 355.02 | 4 | 0.02 | 367.86 | 4 | 0.03 | 344.11 | 3 | 0.01 | 352.70 | 3 | 0.02 | 356.68 | 4 | 0.47 |
| 4 | 375.12 | 5 | 0.02 | 396.11 | 6 | 0.00 | 395.42 | 5 | 0.75 | 375.12 | 5 | 0.00 | 396.11 | 6 | 0.00 | 395.42 | 5 | 0.14 |
| 5 | 373.71 | 3 | 0.01 | 365.55 | 3 | 0.10 | 376.68 | 4 | 0.01 | 373.71 | 3 | 0.01 | 365.55 | 3 | 0.00 | 376.68 | 4 | 0.01 |
| 6 | 432.30 | 4 | 0.00 | 405.99 | 4 | 0.01 | 432.83 | 5 | 0.03 | 432.30 | 4 | 0.02 | 405.99 | 4 | 0.01 | 432.83 | 5 | 0.03 |
| 7 | 691.85 | 4 | 0.01 | 693.58 | 4 | 0.00 | 674.70 | 4 | 0.01 | 691.85 | 4 | 0.03 | 693.46 | 3 | 4.47 | 654.42 | 4 | 4.77 |
| 8 | 718.89 | 3 | 0.11 | 703.67 | 4 | 0.00 | 713.49 | 4 | 0.18 | 718.89 | 3 | 0.10 | 703.67 | 4 | 0.05 | 675.93 | 4 | 0.01 |
| 9 | 494.03 | 5 | 2.61 | 530.35 | 6 | 0.27 | 573.06 | 8 | 0.01 | 494.03 | 5 | 2.61 | 526.48 | 6 | 0.01 | 571.75 | 7 | 0.17 |
| 10 | 536.29 | 3 | 14.68 | 576.50 | 4 | 4.33 | 613.95 | 4 | 8.29 | 521.00 | 3 | 2.86 | 554.57 | 3 | 0.68 | 613.95 | 4 | 0.24 |
| 11 | 581.42 | 4 | 6.22 | 565.46 | 4 | 5.22 | 663.37 | 6 | 2.09 | 562.99 | 4 | 1.51 | 560.43 | 4 | 0.83 | 658.48 | 5 | 1.22 |
| 12 | 471.46 | 6 | 0.00 | 497.63 | 7 | 0.05 | 524.53 | 8 | 0.00 | 471.46 | 6 | 0.00 | 497.63 | 7 | 0.07 | 522.56 | 7 | 0.37 |
| 13 | 2384.40 | 4 | 9.30 | 2239.93 | 4 | 0.35 | 2468.80 | 5 | 40.16 | 2342.09 | 4 | 2.06 | 2208.95 | 3 | 16.71 | 2294.43 | 5 | 1.95 |
| 14 | 878.23 | 4 | 15.98 | 886.27 | 4 | 44.02 | 879.84 | 4 | 51.01 | 784.59 | 3 | 3.50 | 857.36 | 4 | 24.02 | 868.60 | 4 | 34.14 |
| 15 | 853.62 | 3 | 20.42 | 886.93 | 4 | 7.12 | 1024.84 | 5 | 23.10 | 767.08 | 3 | 48.87 | 840.85 | 4 | 14.65 | 998.30 | 5 | 30.86 |
| 16 | 544.24 | 7 | 17.32 | 578.20 | 8 | 4.17 | 610.99 | 9 | 0.11 | 544.24 | 7 | 17.32 | 578.20 | 8 | 4.17 | 610.99 | 9 | 0.08 |
| 17 | 635.94 | 8 | 0.06 | 680.30 | 9 | 0.18 | 723.17 | 12 | 4.70 | 635.94 | 8 | 0.41 | 680.30 | 9 | 0.77 | 722.62 | 12 | 1.08 |
| 18 | 919.65 | 5 | 67.34 | 920.45 | 6 | 85.42 | 971.94 | 7 | 8.75 | 865.67 | 5 | 6.86 | 888.28 | 6 | 4.22 | 965.96 | 7 | 5.55 |
| 19 | 655.97 | 6 | 35.19 | 712.12 | 8 | 9.56 | 742.96 | 9 | 20.53 | 639.40 | 6 | 54.09 | 672.78 | 7 | 45.96 | 714.81 | 8 | 86.22 |
| 20 | 397.05 | 9 | 2.33 | 424.19 | 10 | 75.11 | 466.63 | 12 | 62.67 | 383.65 | 8 | 69.04 | 414.19 | 10 | 70.67 | 448.55 | 11 | 59.09 |
| 21 | 892.75 | 9 | 85.88 | 973.94 | 12 | 27.67 | 1035.99 | 13 | 51.30 | 842.31 | 8 | 115.12 | 922.14 | 11 | 116.98 | 970.50 | 13 | 78.35 |
| 22 | 862.65 | 9 | 44.82 | 935.45 | 11 | 31.69 | 968.10 | 12 | 134.53 | 841.57 | 8 | 12.61 | 897.24 | 11 | 30.78 | 938.27 | 12 | 134.03 |
| 23 | 860.55 | 9 | 30.76 | 955.00 | 12 | 94.76 | 986.13 | 13 | 10.50 | 837.07 | 8 | 64.80 | 906.30 | 11 | 142.12 | 945.58 | 12 | 82.84 |
| 24 | 890.40 | 9 | 36.03 | 948.72 | 12 | 8.44 | 999.72 | 13 | 69.61 | 850.98 | 9 | 78.47 | 912.60 | 11 | 139.84 | 953.55 | 12 | 57.59 |
| 25 | 1102.54 | 12 | 81.73 | 1141.33 | 14 | 199.77 | 1255.11 | 17 | 170.31 | 1051.22 | 11 | 144.49 | 1098.11 | 13 | 151.16 | 1179.71 | 16 | 92.85 |
| 26 | 1039.09 | 11 | 177.26 | 1086.15 | 13 | 147.74 | 1229.65 | 15 | 192.96 | 953.12 | 10 | 111.76 | 1024.31 | 12 | 147.99 | 1179.77 | 14 | 146.04 |
| 27 | 1089.58 | 12 | 86.97 | 1180.91 | 15 | 169.00 | 1283.22 | 17 | 162.44 | 1055.65 | 11 | 69.41 | 1116.42 | 14 | 192.88 | 1226.13 | 17 | 53.29 |
| 28 | 1801.48 | 14 | 149.40 | 2029.48 | 17 | 248.39 | 2334.18 | 20 | 231.19 | 1690.30 | 12 | 224.24 | 1878.15 | 16 | 236.06 | 2192.05 | 18 | 229.74 |
| 29 | 1638.68 | 15 | 245.21 | 1722.74 | 17 | 221.84 | 1986.48 | 21 | 244.40 | 1539.33 | 13 | 248.39 | 1661.51 | 15 | 176.56 | 1878.23 | 19 | 230.14 |
| 30 | 1396.25 | 17 | 271.84 | 1554.94 | 22 | 263.01 | 1663.83 | 25 | 254.09 | 1339.90 | 16 | 188.49 | 1470.26 | 20 | 263.09 | 1611.83 | 24 | 270.40 |
| 31 | 1698.68 | 22 | 240.84 | 1866.44 | 27 | 210.82 | 2029.25 | 32 | 257.27 | 1614.75 | 20 | 267.57 | 1781.28 | 25 | 209.32 | 1934.67 | 30 | 257.01 |
| 32 | 1679.53 | 22 | 270.55 | 1846.20 | 26 | 266.35 | 2026.01 | 31 | 245.78 | 1608.71 | 20 | 197.05 | 1777.99 | 25 | 254.04 | 1926.61 | 29 | 270.13 |
| 33 | 1715.24 | 22 | 191.94 | 1947.88 | 27 | 273.54 | 2191.51 | 33 | 274.83 | 1652.67 | 21 | 216.94 | 1847.64 | 26 | 235.62 | 2058.68 | 31 | 264.21 |
| 34 | 908.90 | 28 | 261.96 | 988.73 | 35 | 291.95 | 1087.44 | 40 | 269.66 | 868.20 | 27 | 242.19 | 924.61 | 32 | 290.76 | 1031.75 | 37 | 299.45 |
| 35 | 1020.11 | 27 | 245.65 | 1156.39 | 34 | 284.95 | 1308.31 | 40 | 245.42 | 956.36 | 25 | 261.80 | 1094.68 | 31 | 295.11 | 1212.11 | 37 | 289.85 |
| 36 | 1126.35 | 27 | 251.71 | 1315.68 | 34 | 289.87 | 1494.50 | 39 | 298.95 | 1046.33 | 25 | 295.39 | 1238.56 | 32 | 292.29 | 1422.05 | 37 | 289.13 |

Table A.17: Results on the 36 instances of the 2L-VRPB – Class 4

| # | 2\|SO\|L | | | | | | | | | 2\|SR\|L | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Line/Back: 1/1 | | | Line/Back: 2/1 | | | Line/Back: 4/1 | | | Line/Back: 1/1 | | | Line/Back: 2/1 | | | Line/Back: 4/1 | | |
| | BR-LNS | K | t(s) | BR-LNS | K | t(s) | BR-LNS | K | t(s) | BR-LNS | K | t(s) | BR-LNS | K | t(s) | BR-LNS | K | t(s) |
| 1 | 312.12 | 2 | 8.53 | 274.25 | 3 | 0.00 | 275.25 | 3 | 1.50 | 312.12 | 2 | 3.48 | 274.25 | 3 | 0.00 | 259.97 | 3 | 0.00 |
| 2 | 308.76 | 3 | 0.01 | 323.52 | 5 | 0.00 | 299.64 | 5 | 0.00 | 308.76 | 3 | 0.01 | 323.52 | 5 | 0.00 | 299.64 | 5 | 0.00 |
| 3 | 335.54 | 3 | 0.01 | 352.83 | 3 | 2.61 | 356.76 | 4 | 0.04 | 335.54 | 3 | 0.00 | 352.83 | 3 | 0.04 | 356.76 | 4 | 0.01 |
| 4 | 375.12 | 5 | 0.00 | 396.11 | 6 | 0.00 | 410.20 | 6 | 0.01 | 375.12 | 5 | 0.00 | 396.11 | 6 | 0.01 | 410.20 | 6 | 0.02 |
| 5 | 372.12 | 3 | 0.02 | 373.66 | 3 | 0.03 | 385.74 | 4 | 0.01 | 372.12 | 3 | 0.03 | 370.07 | 3 | 0.17 | 385.74 | 4 | 0.00 |
| 6 | 432.30 | 4 | 0.02 | 425.35 | 4 | 0.02 | 432.83 | 5 | 0.06 | 432.30 | 4 | 0.00 | 421.56 | 4 | 1.02 | 432.83 | 5 | 0.02 |
| 7 | 699.27 | 4 | 0.00 | 703.67 | 4 | 0.00 | 674.28 | 3 | 0.73 | 699.27 | 4 | 0.00 | 703.19 | 3 | 1.70 | 631.28 | 3 | 0.17 |
| 8 | 692.26 | 4 | 0.24 | 703.67 | 4 | 0.10 | 660.95 | 4 | 0.01 | 692.26 | 4 | 0.03 | 703.67 | 4 | 0.06 | 654.42 | 4 | 1.36 |
| 9 | 500.57 | 5 | 1.39 | 526.48 | 6 | 0.02 | 571.75 | 7 | 0.41 | 500.57 | 5 | 0.42 | 526.48 | 6 | 0.01 | 571.75 | 7 | 0.03 |
| 10 | 589.43 | 4 | 18.85 | 637.45 | 5 | 17.60 | 663.73 | 5 | 5.52 | 576.93 | 3 | 3.42 | 637.45 | 4 | 17.60 | 657.26 | 5 | 2.74 |
| 11 | 644.27 | 4 | 2.46 | 638.45 | 5 | 11.77 | 737.89 | 6 | 0.34 | 626.79 | 4 | 0.07 | 612.41 | 5 | 1.39 | 696.65 | 5 | 18.11 |
| 12 | 475.76 | 6 | 0.06 | 498.25 | 7 | 0.17 | 534.87 | 8 | 5.43 | 475.76 | 6 | 0.06 | 498.25 | 7 | 0.15 | 522.56 | 7 | 21.80 |
| 13 | 2354.57 | 4 | 10.49 | 2478.79 | 5 | 10.86 | 2518.66 | 6 | 1.46 | 2354.57 | 4 | 9.02 | 2439.28 | 5 | 9.53 | 2462.42 | 5 | 7.15 |
| 14 | 777.60 | 3 | 48.01 | 871.59 | 4 | 2.67 | 900.65 | 4 | 38.14 | 767.48 | 3 | 9.30 | 838.18 | 4 | 35.46 | 888.48 | 4 | 42.69 |
| 15 | 909.02 | 4 | 26.86 | 899.68 | 4 | 37.76 | 1085.14 | 6 | 31.10 | 895.65 | 4 | 12.24 | 887.81 | 4 | 13.53 | 1011.12 | 5 | 50.55 |
| 16 | 543.39 | 7 | 0.11 | 599.82 | 8 | 0.15 | 622.18 | 9 | 0.70 | 542.60 | 7 | 0.11 | 582.21 | 8 | 0.68 | 609.85 | 9 | 42.57 |
| 17 | 638.14 | 9 | 0.21 | 680.30 | 9 | 0.50 | 724.47 | 12 | 0.43 | 635.94 | 8 | 0.15 | 680.30 | 9 | 0.09 | 723.17 | 12 | 2.92 |
| 18 | 918.57 | 6 | 16.46 | 963.98 | 6 | 11.90 | 989.86 | 7 | 86.07 | 899.17 | 5 | 35.95 | 940.67 | 6 | 17.23 | 982.23 | 7 | 4.34 |
| 19 | 637.33 | 6 | 78.63 | 678.47 | 7 | 85.96 | 722.10 | 9 | 68.21 | 633.98 | 6 | 10.03 | 661.86 | 7 | 4.34 | 710.00 | 9 | 13.09 |
| 20 | 398.26 | 9 | 32.33 | 467.63 | 11 | 59.72 | 500.80 | 12 | 97.53 | 385.51 | 8 | 55.95 | 446.46 | 10 | 86.71 | 489.69 | 12 | 96.52 |
| 21 | 844.89 | 9 | 58.94 | 860.51 | 9 | 117.28 | 906.63 | 11 | 131.54 | 824.78 | 8 | 75.17 | 843.69 | 9 | 38.61 | 878.72 | 10 | 89.63 |
| 22 | 899.02 | 10 | 64.36 | 875.77 | 11 | 88.49 | 996.26 | 14 | 143.92 | 853.02 | 9 | 124.66 | 863.08 | 10 | 142.58 | 976.26 | 13 | 127.74 |
| 23 | 862.22 | 9 | 146.63 | 913.78 | 11 | 124.22 | 956.42 | 12 | 118.70 | 830.68 | 8 | 106.66 | 895.69 | 10 | 90.36 | 933.62 | 12 | 61.56 |
| 24 | 896.58 | 9 | 71.11 | 948.54 | 11 | 148.51 | 1002.35 | 13 | 122.82 | 868.43 | 9 | 78.26 | 917.92 | 11 | 146.26 | 980.42 | 13 | 56.83 |
| 25 | 1091.96 | 12 | 142.24 | 1178.43 | 14 | 142.52 | 1271.21 | 17 | 168.07 | 1068.36 | 11 | 135.90 | 1129.62 | 13 | 197.09 | 1240.41 | 17 | 161.91 |
| 26 | 1096.63 | 12 | 184.96 | 1107.94 | 14 | 162.72 | 1283.55 | 17 | 79.58 | 1033.00 | 11 | 106.71 | 1075.59 | 13 | 68.33 | 1259.88 | 16 | 169.04 |
| 27 | 1058.67 | 11 | 175.05 | 1091.93 | 13 | 158.80 | 1207.54 | 16 | 176.60 | 1013.58 | 10 | 165.58 | 1071.70 | 12 | 126.30 | 1172.75 | 15 | 103.15 |
| 28 | 1813.13 | 13 | 233.55 | 1974.14 | 16 | 224.04 | 2186.90 | 18 | 245.29 | 1751.68 | 12 | 240.19 | 1914.67 | 16 | 225.98 | 2103.46 | 18 | 245.10 |
| 29 | 1667.36 | 15 | 163.74 | 1881.43 | 20 | 244.90 | 1905.98 | 22 | 249.37 | 1634.34 | 14 | 248.95 | 1840.32 | 19 | 246.56 | 1854.41 | 21 | 231.39 |
| 30 | 1385.71 | 17 | 51.51 | 1566.45 | 21 | 248.90 | 1642.08 | 24 | 260.73 | 1341.79 | 16 | 92.31 | 1504.83 | 20 | 272.04 | 1570.02 | 23 | 255.90 |
| 31 | 1730.54 | 23 | 265.37 | 1934.79 | 28 | 222.73 | 2141.65 | 34 | 273.80 | 1677.90 | 22 | 198.52 | 1880.64 | 27 | 260.57 | 2042.84 | 32 | 269.36 |
| 32 | 1687.62 | 21 | 230.04 | 1875.74 | 27 | 262.11 | 2005.18 | 30 | 270.62 | 1623.55 | 20 | 267.73 | 1815.40 | 25 | 269.48 | 1940.05 | 29 | 240.33 |
| 33 | 1732.86 | 23 | 271.81 | 1950.56 | 28 | 247.95 | 2147.33 | 32 | 273.46 | 1663.38 | 21 | 256.49 | 1880.26 | 27 | 260.27 | 2053.65 | 31 | 257.33 |
| 34 | 877.18 | 28 | 249.25 | 953.31 | 33 | 293.35 | 1078.34 | 40 | 298.62 | 855.08 | 26 | 277.03 | 921.45 | 32 | 242.92 | 1047.32 | 39 | 254.37 |
| 35 | 1027.38 | 28 | 296.90 | 1205.36 | 36 | 285.63 | 1318.93 | 41 | 255.82 | 982.16 | 26 | 202.87 | 1181.60 | 35 | 216.55 | 1259.19 | 39 | 287.19 |
| 36 | 1052.64 | 25 | 299.11 | 1230.27 | 32 | 280.14 | 1448.06 | 38 | 294.29 | 997.65 | 24 | 272.05 | 1182.88 | 31 | 294.00 | 1376.93 | 37 | 298.79 |

Table A.18: Results on the 36 instances of the 2L-VRPB – Class 5

| # | 2\|SO\|L | | | | | | | | | 2\|SR\|L | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Line/Back: 1/1 | | | Line/Back: 2/1 | | | Line/Back: 4/1 | | | Line/Back: 1/1 | | | Line/Back: 2/1 | | | Line/Back: 4/1 | | |
| | BR-LNS | K | t(s) | BR-LNS | K | t(s) | BR-LNS | K | t(s) | BR-LNS | K | t(s) | BR-LNS | K | t(s) | BR-LNS | K | t(s) |
| 1 | 307.63 | 3 | 0.02 | 284.22 | 3 | 0.17 | 259.97 | 3 | 0.01 | 301.99 | 2 | 5.94 | 274.25 | 3 | 0.00 | 259.97 | 3 | 0.01 |
| 2 | 308.76 | 3 | 0.00 | 323.52 | 5 | 0.00 | 299.64 | 5 | 0.00 | 308.76 | 3 | 0.01 | 323.52 | 5 | 0.00 | 299.64 | 5 | 0.00 |
| 3 | 335.54 | 3 | 0.01 | 352.70 | 3 | 0.09 | 349.12 | 4 | 0.00 | 335.54 | 3 | 0.00 | 352.70 | 3 | 0.05 | 349.12 | 4 | 0.01 |
| 4 | 375.12 | 5 | 0.00 | 396.11 | 6 | 0.00 | 395.42 | 5 | 0.01 | 375.12 | 5 | 0.00 | 396.11 | 6 | 0.00 | 395.42 | 5 | 0.02 |
| 5 | 372.12 | 3 | 0.04 | 365.55 | 3 | 0.00 | 376.68 | 4 | 0.01 | 372.12 | 3 | 0.04 | 365.55 | 3 | 0.00 | 376.68 | 4 | 0.01 |
| 6 | 432.30 | 4 | 0.01 | 405.99 | 4 | 0.00 | 432.83 | 5 | 0.01 | 432.30 | 4 | 0.01 | 405.99 | 4 | 0.00 | 432.83 | 5 | 0.01 |
| 7 | 689.32 | 3 | 3.70 | 693.58 | 4 | 0.00 | 631.28 | 3 | 3.62 | 689.32 | 3 | 1.39 | 693.58 | 4 | 0.02 | 631.28 | 3 | 0.33 |
| 8 | 677.52 | 3 | 3.47 | 693.58 | 4 | 0.00 | 603.43 | 3 | 0.73 | 677.52 | 3 | 0.18 | 693.58 | 4 | 0.00 | 600.65 | 3 | 0.38 |
| 9 | 494.03 | 5 | 9.49 | 526.48 | 6 | 0.01 | 571.75 | 7 | 0.09 | 494.03 | 5 | 19.62 | 526.48 | 6 | 0.04 | 571.75 | 7 | 0.01 |
| 10 | 571.68 | 4 | 23.03 | 590.46 | 4 | 0.81 | 609.63 | 5 | 1.04 | 568.53 | 3 | 25.50 | 590.46 | 4 | 0.44 | 602.65 | 4 | 25.58 |
| 11 | 573.31 | 4 | 26.99 | 565.82 | 4 | 27.80 | 614.38 | 5 | 4.37 | 571.68 | 4 | 7.49 | 563.95 | 4 | 1.05 | 612.30 | 5 | 0.99 |
| 12 | 471.46 | 6 | 0.00 | 497.63 | 7 | 0.08 | 522.56 | 7 | 0.40 | 471.46 | 6 | 0.00 | 497.63 | 7 | 0.05 | 522.56 | 7 | 0.21 |
| 13 | 2326.80 | 4 | 37.73 | 2329.21 | 4 | 44.17 | 2286.38 | 5 | 39.29 | 2326.80 | 4 | 0.52 | 2300.45 | 4 | 35.77 | 2251.06 | 4 | 18.58 |
| 14 | 771.31 | 3 | 31.59 | 767.04 | 3 | 7.69 | 863.12 | 4 | 30.93 | 769.21 | 3 | 5.79 | 756.84 | 3 | 11.87 | 863.12 | 4 | 30.93 |
| 15 | 907.13 | 4 | 22.48 | 892.28 | 4 | 44.36 | 1002.07 | 5 | 8.00 | 901.79 | 4 | 31.78 | 884.92 | 4 | 4.08 | 1000.43 | 5 | 2.80 |
| 16 | 542.60 | 7 | 0.17 | 578.20 | 8 | 0.01 | 610.99 | 9 | 0.10 | 542.60 | 7 | 0.18 | 578.20 | 8 | 2.72 | 609.85 | 9 | 0.14 |
| 17 | 635.94 | 8 | 0.15 | 680.30 | 9 | 0.15 | 722.62 | 12 | 0.37 | 635.94 | 8 | 0.26 | 680.30 | 9 | 0.56 | 722.62 | 12 | 0.10 |
| 18 | 845.35 | 4 | 21.74 | 880.82 | 5 | 50.06 | 909.63 | 6 | 81.39 | 845.35 | 4 | 21.74 | 880.82 | 5 | 7.35 | 896.74 | 6 | 79.67 |
| 19 | 617.50 | 6 | 0.16 | 625.08 | 6 | 3.38 | 637.06 | 7 | 68.32 | 604.80 | 5 | 13.21 | 617.53 | 6 | 80.93 | 636.34 | 7 | 39.16 |
| 20 | 375.20 | 7 | 95.47 | 392.19 | 9 | 85.17 | 445.67 | 11 | 64.52 | 372.24 | 7 | 71.25 | 390.85 | 9 | 39.54 | 435.31 | 10 | 62.31 |
| 21 | 783.33 | 7 | 95.90 | 815.47 | 9 | 17.72 | 848.91 | 10 | 137.90 | 774.43 | 7 | 60.19 | 794.37 | 8 | 119.62 | 828.99 | 9 | 140.46 |
| 22 | 805.17 | 7 | 23.92 | 843.17 | 9 | 98.33 | 888.16 | 11 | 92.15 | 792.44 | 7 | 59.95 | 838.64 | 9 | 31.77 | 880.79 | 10 | 79.48 |
| 23 | 802.86 | 8 | 20.87 | 841.20 | 9 | 132.62 | 873.26 | 10 | 132.09 | 798.15 | 7 | 28.19 | 819.20 | 9 | 69.03 | 866.27 | 10 | 148.65 |
| 24 | 844.15 | 8 | 146.21 | 884.62 | 10 | 101.70 | 922.58 | 12 | 140.82 | 834.08 | 8 | 16.70 | 881.49 | 10 | 115.05 | 912.80 | 11 | 132.09 |
| 25 | 984.61 | 9 | 74.50 | 1021.60 | 11 | 161.28 | 1088.57 | 13 | 121.26 | 983.36 | 9 | 105.50 | 1004.54 | 11 | 160.07 | 1079.72 | 13 | 100.11 |
| 26 | 903.86 | 10 | 175.61 | 952.00 | 12 | 158.60 | 1125.75 | 14 | 186.52 | 899.37 | 10 | 166.18 | 923.19 | 11 | 118.88 | 1099.40 | 13 | 181.31 |
| 27 | 1011.70 | 10 | 142.89 | 1074.09 | 12 | 67.72 | 1149.52 | 14 | 151.64 | 1009.33 | 10 | 41.19 | 1050.65 | 11 | 124.38 | 1138.12 | 14 | 174.58 |
| 28 | 1616.89 | 11 | 187.81 | 1827.23 | 14 | 247.16 | 2047.11 | 16 | 226.90 | 1595.35 | 11 | 244.72 | 1774.85 | 14 | 246.57 | 2007.87 | 16 | 155.53 |
| 29 | 1625.58 | 13 | 186.08 | 1798.12 | 16 | 245.92 | 1935.20 | 20 | 248.51 | 1625.08 | 13 | 248.21 | 1764.25 | 16 | 240.43 | 1881.82 | 19 | 234.08 |
| 30 | 1236.57 | 13 | 229.24 | 1326.03 | 16 | 188.49 | 1445.86 | 19 | 215.40 | 1209.27 | 13 | 215.14 | 1284.32 | 15 | 260.08 | 1406.47 | 19 | 195.55 |
| 31 | 1545.89 | 19 | 254.49 | 1719.53 | 23 | 272.04 | 1864.63 | 28 | 153.84 | 1513.71 | 18 | 273.25 | 1688.12 | 22 | 244.31 | 1812.71 | 26 | 258.41 |
| 32 | 1521.70 | 18 | 183.21 | 1662.50 | 21 | 274.11 | 1795.95 | 26 | 250.65 | 1503.93 | 18 | 273.31 | 1624.16 | 21 | 244.56 | 1757.09 | 25 | 267.40 |
| 33 | 1505.30 | 18 | 270.56 | 1663.11 | 22 | 274.45 | 1880.20 | 28 | 219.55 | 1496.67 | 18 | 202.26 | 1628.61 | 21 | 258.68 | 1836.32 | 27 | 271.03 |
| 34 | 808.02 | 24 | 247.37 | 867.07 | 29 | 299.55 | 946.10 | 33 | 297.71 | 794.92 | 23 | 270.59 | 849.22 | 28 | 290.04 | 932.51 | 32 | 295.33 |
| 35 | 893.69 | 22 | 264.60 | 1026.55 | 29 | 256.81 | 1146.00 | 34 | 187.40 | 876.87 | 22 | 277.45 | 1011.02 | 28 | 291.24 | 1115.01 | 32 | 263.24 |
| 36 | 946.13 | 22 | 265.28 | 1109.22 | 28 | 289.06 | 1309.89 | 34 | 295.62 | 926.31 | 22 | 282.21 | 1087.75 | 27 | 279.59 | 1281.93 | 33 | 287.67 |